Midterm Exam 1 is scheduled for Monday, January 25th, at 3:00pm (in class). Remember that calculators are not allowed for this exam. The following exercises are just a sample of what may be on the exam. Do not restrict your studying to this review, and it is no guarantee that everything on this review will be on the midterm. These exercises are all examples of what knowledge/skills you should have gotten from the material covered so far. Other very important resources for studying include the WebWork assignments, lecture guides and exercises in the text.

1. The Concept Check, True-False Quiz, and Exercises in the Chapter 2 Review of the textbook. You can ignore references to the Intermediate Value Theorem (we'll look at this later in the quarter).

2. Using the definition of the derivative, compute $f'''(x)$ for $f(x) = 2x^2 - 4$.

3. Identify the values of a and b for which

$$f(x) = \begin{cases} x^3 + 2x - a & x < -1 \\ ax + b & x \geq -1 \end{cases}$$

   is (a) continuous, then (b) differentiable on $(-\infty, \infty)$.

4. Find each limit using algebra and/or limit laws, or state that it does not exist.

   (a) $\displaystyle\lim_{t \to -\infty} \frac{t^4 + 1}{t^5 + 3t + 4}$

   (b) $\displaystyle\lim_{x \to 2} \frac{\sqrt{x + 2} - 2}{x - 2}$

   (c) $\displaystyle\lim_{b \to 1} \log\left(e^{b^2 - 1} + 3^{b^2 + 1}\right)$

5. Find all asymptotes of $f(x) = \dfrac{4 - x^2}{2x^2 - x - 6}$. Use limits to support your conclusion.

6. Consider the graph of $y = f(x)$ given at the right.

   (a) Find $\displaystyle\lim_{x \to 1} f(x)$, or state that it does not exist.

   (b) Find $\displaystyle\lim_{x \to 3} f(x)$, or state that it does not exist.

   (c) Find $\displaystyle\lim_{x \to -2^-} f(x)$, or state that it does not exist.

   (d) Find $\displaystyle\lim_{x \to -2^+} f(x)$, or state that it does not exist.

   (e) On what interval(s) is the function depicted continuous? (Assume that a complete graph has been given)

   (f) On what interval(s) is the function depicted differentiable?

   (g) On what interval(s) is $f'(x)$ positive?

   (h) For which value(s) of x is $f'(x) = 0$?

   (i) On what interval(s) is $f''(x)$ positive?
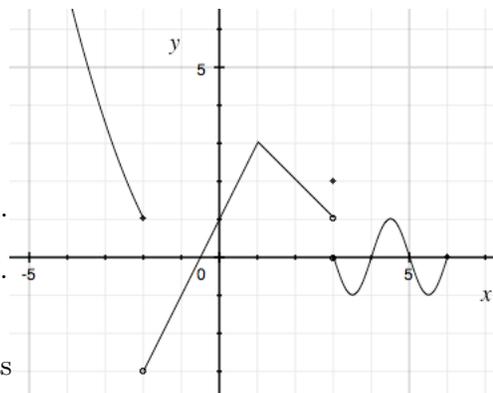
7. Write the equation of the tangent line to $y = \dfrac{3}{x+1}$ at $x = 4$.

8. A 1-kg rocket is fired from the surface of the earth, its height is given by h and gravitational force exerted between the Earth and the rocket is given by $F(h) = \dfrac{GM}{(r+h)^2}$, where G is the gravitational constant, M is the mass of the Earth, and r is the radius of the Earth (all constants). What happens to the force as h goes toward 0? What happens to the force as h goes toward infinity?

9. The height (in meters) of the same rocket from the problem above can be written $h(t) = 11000t - 4.9t^2$ where t is the time (in seconds) since it reached its maximum velocity of 11,000 meters per second.

   (a) What is the rocket's average velocity during the first thirty seconds?

   (b) Find the instantaneous velocity of the rocket after two minutes.

   (c) What is the rocket's acceleration?

10. Determine whether each of the following statements is true or false, and provide an explanation/justification for your decision.

    (a) A continuous function is necessarily differentiable.

    (b) $k'(x) = \dfrac{k(x+h) - k(x)}{h}$

11. Given the graph of $f'(x)$ below, $f(-3) = 0$, and $f(0) = 0$, sketch a graph of $f(x)$.
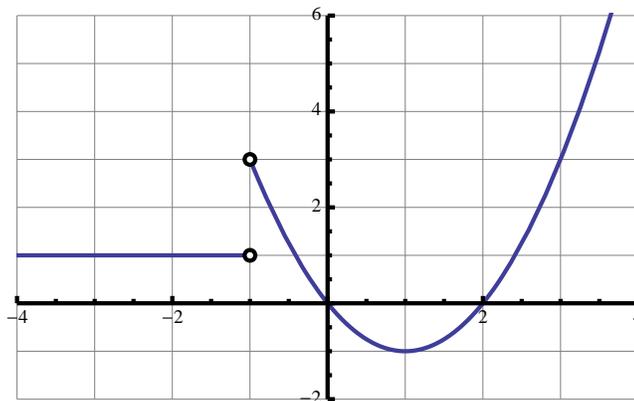
12. Let $f(x) = \sqrt{1-x}$. Where is f differentiable? Using the definition of the derivative, find $f'(x)$.

13. Suppose that f and g are continuous functions. If $f(-1) = 2$ and $\lim\limits_{x \to -1} (f(x) + 2g(x) - x) = 7$, then find $g(-1)$.