Assignment 1. Due Monday, April 4.

Read the preface and pages 1-12 of the notes. Also read pages 3-6 of McCleary, paying particular attention to the proof of proposition 1.2. This is a special case of the Gauss-Bonnet theorem, so the course begins and ends with the same theorem.

1. Give your own version of the proof of proposition 1.2. Try to avoid all technical terms; the proof should make perfect sense to a high school student. Your high school student knows that the area of a sphere of radius one is 4π.
2. Find the length of $\gamma(t) = \Big(2t, t^2, \ln t\Big)$ from $t = 1$ to $t = 2$. Look for a trick which simplifies the integrand.
3. Draw a picture of the curve $\gamma(t) = \Big(e^t \cos t, e^t \sin t, e^t\Big)$.
4. Find a curve $\tau(u)$ which parameterizes the curve $\gamma(t) = \Big(e^t \cos t, e^t \sin t, e^t\Big)$ by arc length.
5. Let $u(t) = \int_{t_0}^{t} ||\gamma'(t)\ dt||$ be the length integral of a curve $\gamma(t)$. This is an abuse of notation because t is used for the variable of integration and also the upper limit of integration; change the notation if you are dislike mine. If $\tau(u)$ represents the same curve parameterized by arc length, then $\tau(u(t)) = \gamma(t)$.

   Recall that $\frac{d\tau}{du} = T$ and $\frac{dT}{du} = \kappa N$ by definition of T, N, and κ. Use the chain rule to obtain the following results:

   (a) $\gamma'(t) = T\frac{du}{dt}$

   (b) $\gamma''(t) = \kappa N \left(\frac{du}{dt}\right)^2 + T\frac{d^2u}{dt^2}$

   (c) $\frac{du}{dt} = ||\gamma'(t)||$

   (d) $\frac{d^2u}{dt^2} = \frac{\gamma' \cdot \gamma''}{||\gamma'||}$

   Using these results, find a formula for κ which depends only on $\gamma(t)$ and thus can be applied when it is impossible to reparameterize by arc length.
6. Use your formula to find the osculating circle to $y = x^2$ at $x = 1$. Draw a picture. (Incidentally, if you know how to use Mathematica, draw an accurate picture to convince yourself that your osculating circle is indeed the best approximating circle.)