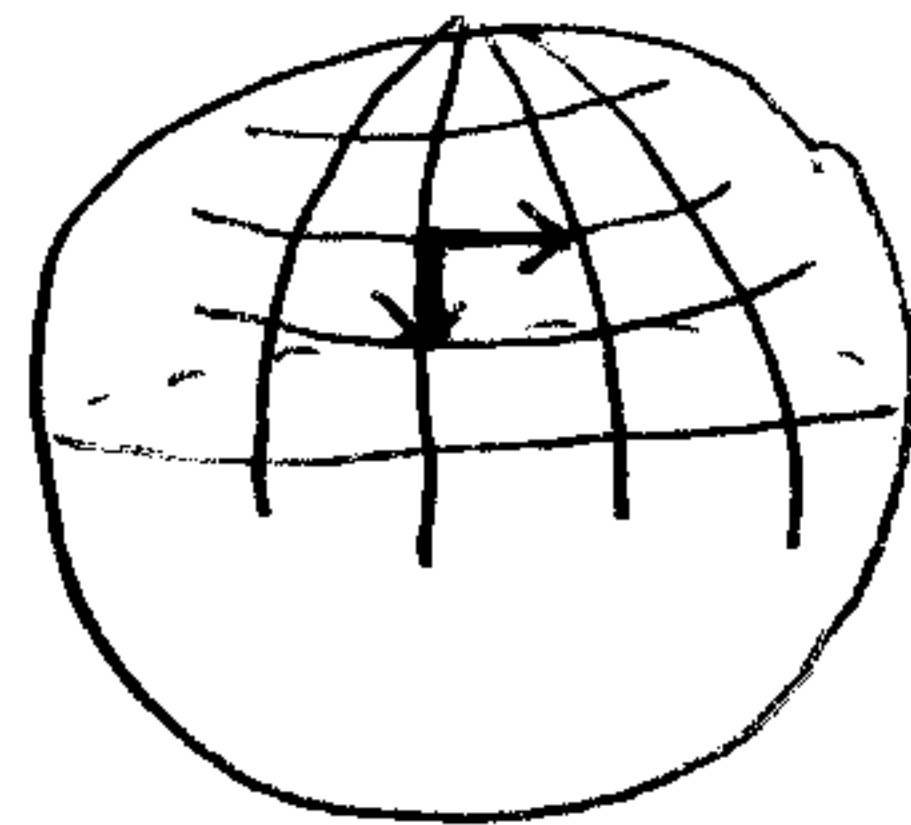
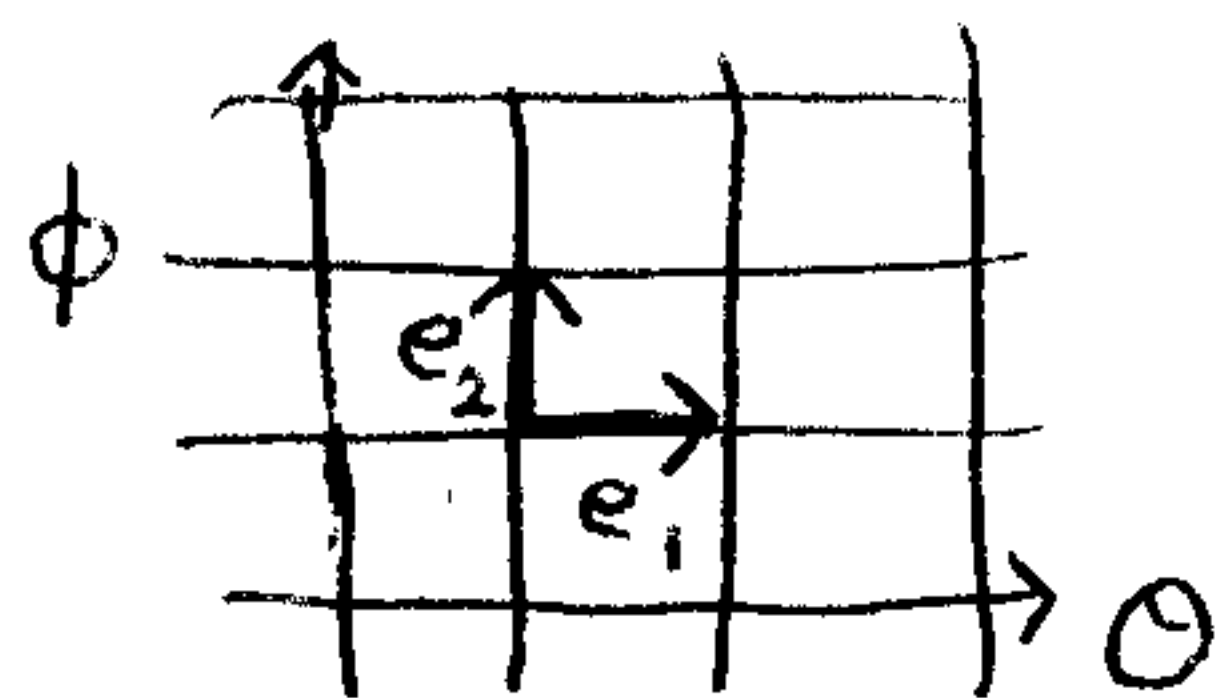


The Entire Theory in the Case of the Sphere

Consider the sphere with spherical coordinates $(\theta, \phi) \rightarrow (\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi)$. Spherical coordinates differ from standard longitude and latitude conventions in one small way: increasing ϕ moves down toward the equator rather than up toward the north pole.



Standard Formulas

$$e_1 = (1, 0) = \frac{\partial}{\partial \theta} \leftrightarrow \frac{\partial s}{\partial \theta} = (-\sin \phi \sin \theta, \sin \phi \cos \theta, 0)$$

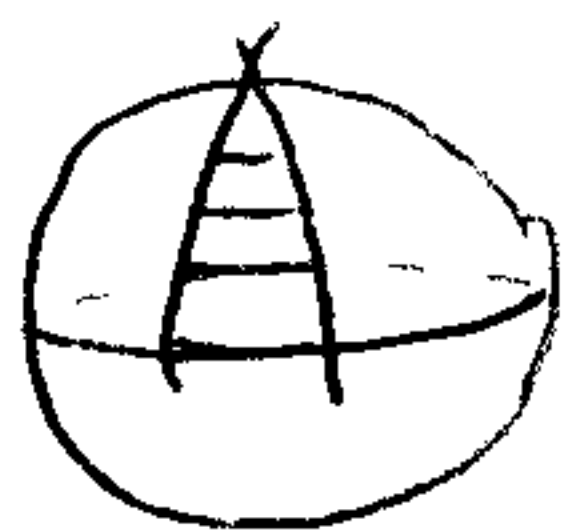
$$e_2 = (0, 1) = \frac{\partial}{\partial \phi} \leftrightarrow \frac{\partial s}{\partial \phi} = (\cos \phi \cos \theta, \cos \phi \sin \theta, -\sin \phi)$$

$$n = (\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi)$$

$$g_{11} = \sin^2 \phi$$

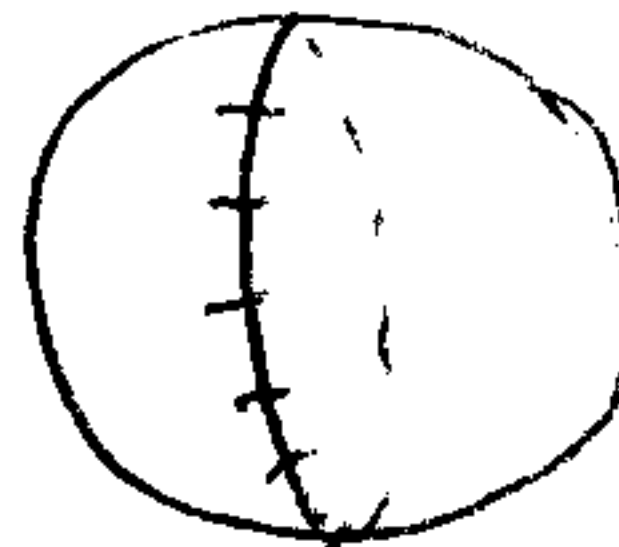
$$g_{12} = 0$$

$$g_{22} = 1$$



horizontal distances get shorter

longitude and latitude always perpendicular



these distances are all equal

Derivatives of Basis Vector Fields

$$e_1(e_1) = \frac{\partial}{\partial \theta} (-\sin \phi \sin \theta, \sin \phi \cos \theta, 0) = (-\sin \phi \cos \theta, -\sin \phi \sin \theta, 0)$$

$$e_1(e_2) = (-\cos \phi \sin \theta, \cos \phi \cos \theta, 0)$$

$$e_2(e_1) = (-\cos \phi \sin \theta, \cos \phi \cos \theta, 0)$$

$$e_2(e_2) = (-\sin \phi \cos \theta, -\sin \phi \sin \theta, -\cos \phi)$$

Decomposition: Now we decompose these derivatives into normal and tangential components. Notice that the basic equation below implies that $b(X, Y) = X(Y) \cdot n$.

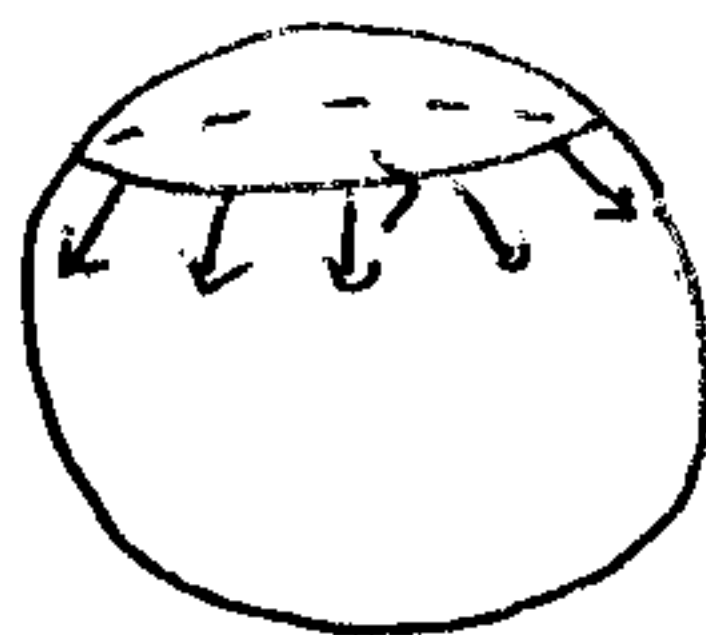
$$X(Y) = b(X, Y)n + \nabla_X Y.$$

$$b(e_1, e_1) = (-\sin \phi \cos \theta, -\sin \phi \sin \theta, 0) \cdot (\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi) = -\sin^2 \phi$$

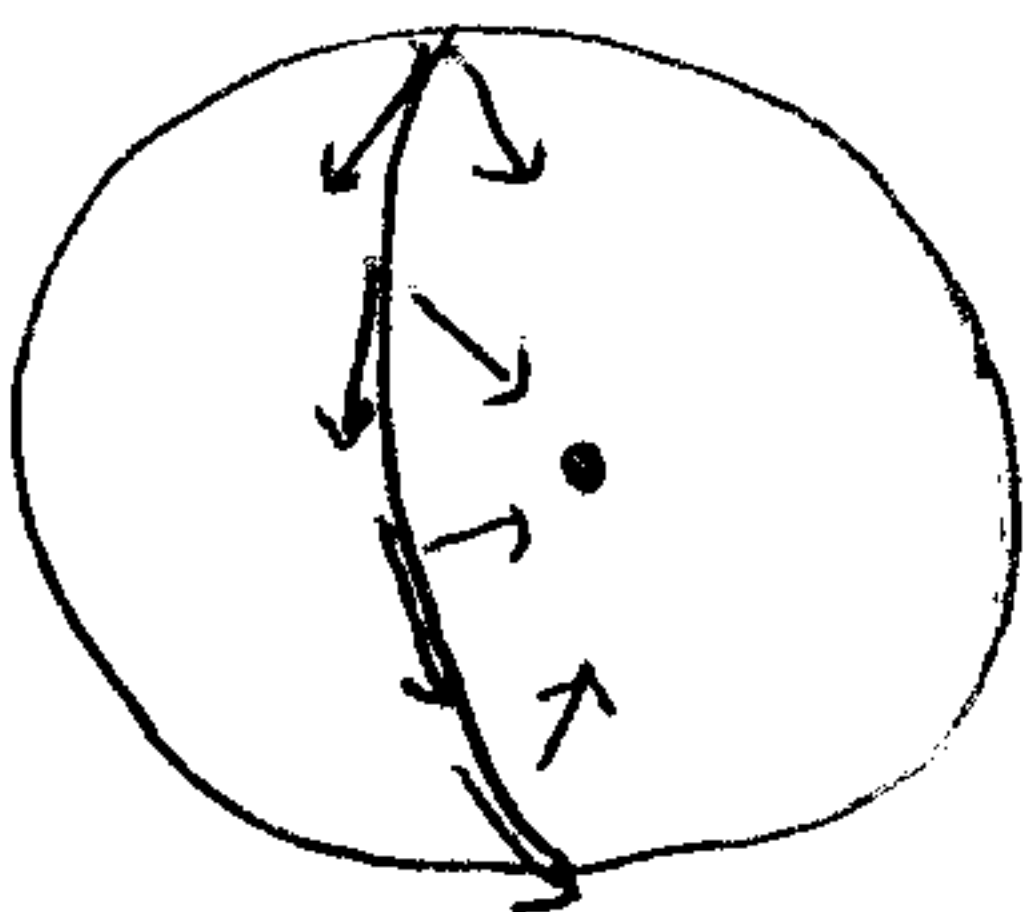
$$b(e_1, e_2) = 0$$

$$b(e_2, e_1) = 0$$

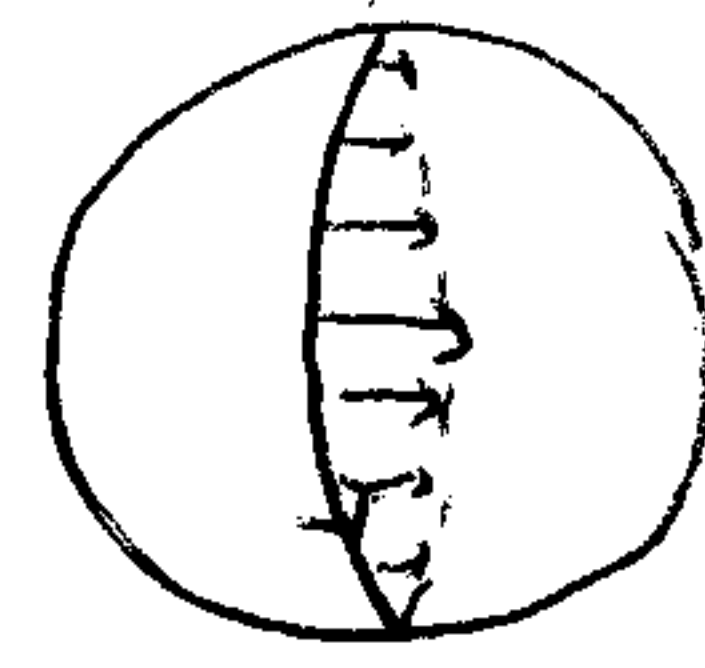
$$b(e_2, e_2) = -1$$



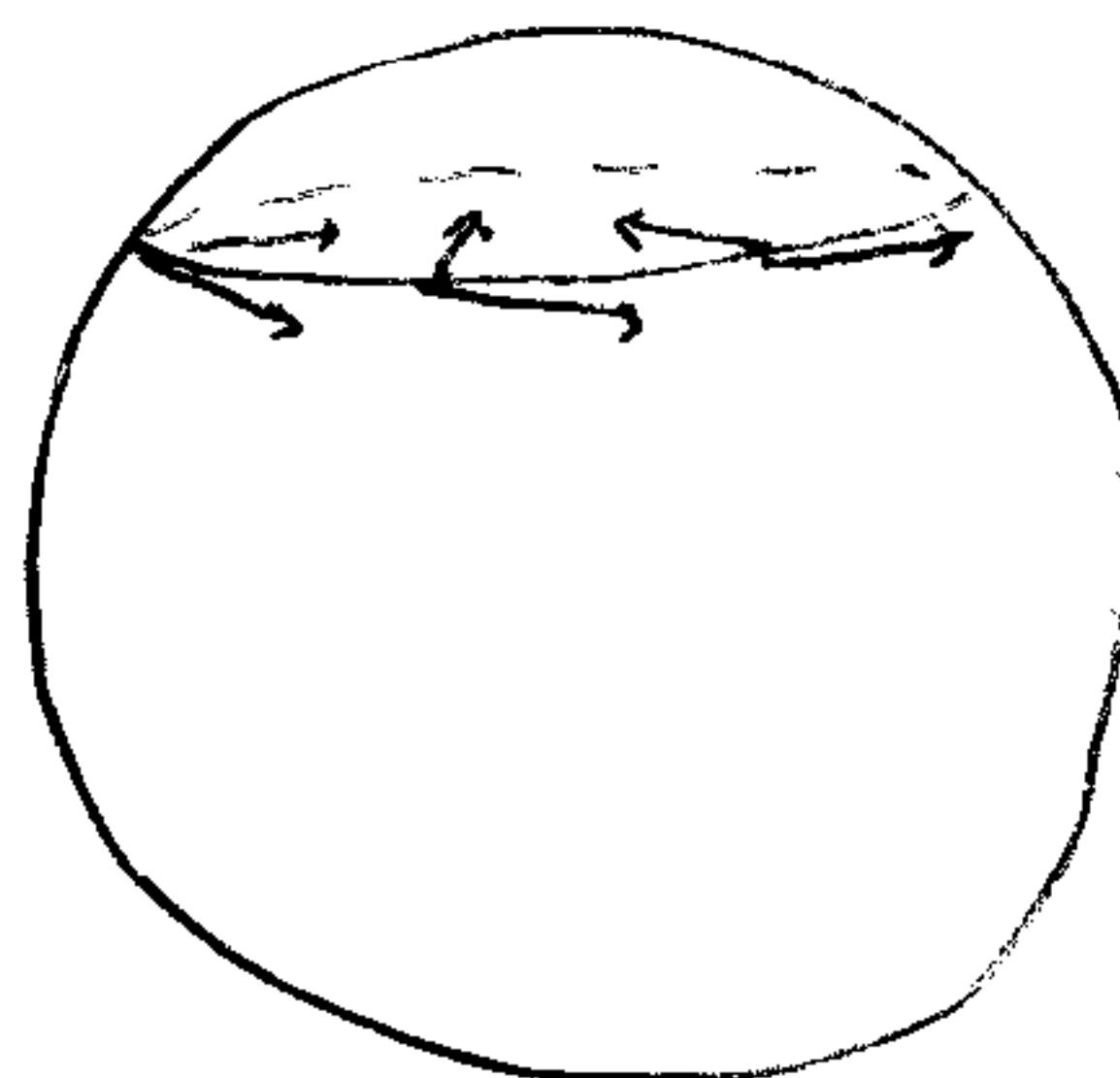
these derivatives are all normal of constant length



and



these derivatives are tangential with no normal component



these derivatives have both normal and tangential components. The normal component is largest at the equator and vanishes at the poles