B Matrix: Recall that $b(X,Y) = -< B(X), Y >$ The dot product of $B(X)$ with respect to the basis vectors completely determines $B(X)$, so these formulas can be used to read off $B(e_1)$ and $B(e_2)$.

$< B(e_1), e_1 >= \sin^2\phi$

$< B(e_1), e_2 >= 0$

$< B(e_2), e_1 >= 0$

$< B(e_2, e_2 >= 1$

Principal Curvatures: It follows that $B(e_1) = e_1$ and $B(e_2) = e_2$. (Notice that the term $\sin^2\phi$ comes from the fancy inner product.) So $\kappa_1 = \kappa_2 = -1$. This sign arises because the normal points outward but the principal curvatures are both inward toward the center of the sphere.

Tangential Components: We can read off the tangential components of derivatives using the formula

$$\nabla_X Y = X(Y) - b(X,Y)n$$

$$\nabla_{e_1} e_1 = (-\sin\phi\cos\theta, -\sin\phi\sin\theta, 0) + \sin^2\phi(\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)$$

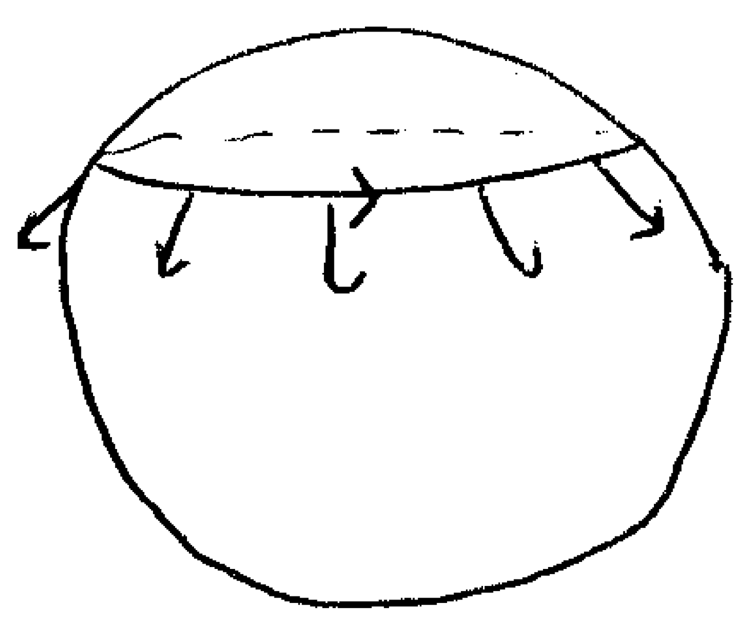
$$= (-\sin\phi\cos\theta\cos^2\phi, -\sin\phi\sin\theta\cos^2\phi, \sin^2\phi\cos\phi) = -\sin\phi\cos\phi(\cos\phi\cos\theta, \cos\phi\sin\theta, -\sin\phi) =$$

$$= -\sin\phi\cos\phi\ e_2$$

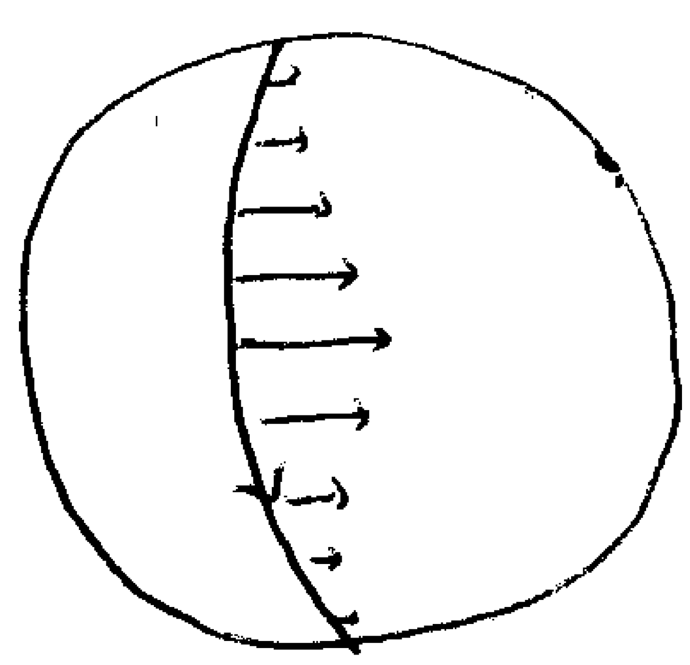
$\nabla_{e_1} e_2 = \frac{\cos\phi}{\sin\phi}\ e_1$

$\nabla_{e_2} e_1 = \frac{\cos\phi}{\sin\phi}\ e_1$

$\nabla_{e_2} e_2 = 0$

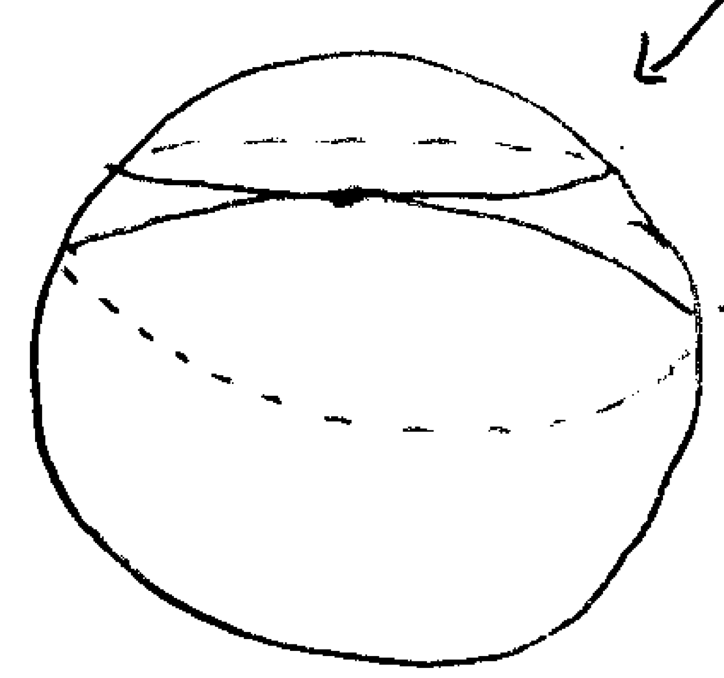


The change in e_2 is tangential pointing in the e_1 direction. There is no change at the equator, but a large change near the poles



The change in e_1 is tangential in the e_1 direction. It is positive in the northern hemisphere and negative in the southern hemisphere

The tangential component of change points in the $-e_2$ direction.



This great circle looks like a straight line. The constant ϕ circle curves upward in the $-e_2$ direction away