Brief Article

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Folks,

Here are the answers to the first two review questions in section six.

Example:

Pulling g back to coordinates gives

$$g(x,y) = xy + (x^2+y^2)^2$$

we have

$$X(g) = 2\frac{\partial g}{\partial x} + 3\frac{\partial g}{\partial y} = 2\left(y + 2(x^2+y^2)2x\right) + 3\left(x + 2(x^2+y^2)2y\right)$$

Evaluating at $(1,1)$ gives $2 \cdot 9 + 3 \cdot 9 = 5 \cdot 9 = 45$.

Example:

To compute the derivative of U, we just write U in coordinates and then differentiate each component separately. So

$$U = (y,x,z) = (y,x,x^2+y^2)$$

and

$$X(U) = \left(\left(2\frac{\partial g}{\partial x} + 3\frac{\partial g}{\partial y}\right)y, \left(2\frac{\partial g}{\partial x} + 3\frac{\partial g}{\partial y}\right)x, \left(2\frac{\partial g}{\partial x} + 3\frac{\partial g}{\partial y}\right)(x^2+y^2)\right)$$

and so

$$X(U) = (3, 2, 4x+6y)$$

Next we need the unit normal. We have

$$N = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right) = (-2x, -2y, 1)$$

and so

$$n = \frac{(-2x, -2y, 1)}{\sqrt{1 + 4x^2 + 4y^2}}$$

If $X(U) = \lambda n + Y$ where λ is a scaler and Y is a tangent vector, then dotting both sides with n gives $X(U) \cdot n = \lambda$. So

$$\lambda = (3, 2, 4x + 6y) \cdot \frac{(-2x, -2y, 1)}{\sqrt{1 + 4x^2 + 4y^2}} = \frac{(-6x - 4y + 4x + 6y)}{\sqrt{1 + 4x^2 + 4y^2}} = \frac{2y - 2x}{\sqrt{1 + 4x^2 + 4y^2}}$$

Finally

$$Y = X(U) - \lambda n = (3, 2, 4x + 6y) - \frac{2y - 2x}{\sqrt{1 + 4x^2 + 4y^2}} \frac{(-2x, -2y, 1)}{\sqrt{1 + 4x^2 + 4y^2}}$$

and so

$$Y = \frac{(3, 2, 4x + 6y)(1 + 4x^2 + 4y^2) + (-2x, -2y, 1)(2x - 2y)}{1 + 4x^2 + 4y^2}$$

or

$$Y = \frac{\left(3 + 8x^2 + 12y^2 + 4xy, 2 + 8x^2 + 12y^2 - 4xy, 6x + 4y + 16x^3 + 24x^2 y + 16xy^2 + 24y^3\right)}{1 + 4x^2 + 4y^2}$$

As a check, note that $\frac{\partial}{\partial x}$ corresponds to the three dimensional vector $(1, 0, 2x)$ and $\frac{\partial}{\partial y}$ corresponds to the three dimensional vector $(0, 1, 2y)$. If this Y is really tangent, then it can be written

$$Y = \frac{3 + 8x^2 + 12y^2 + 4xy}{1 + 4x^2 + 4y^2} (1, 0, 2x) + \frac{2 + 8x^2 + 12y^2 - 4xy}{1 + 4x^2 + 4y^2} (0, 1, 2y)$$

You can check that this indeed gives the correct third coordinate.