

# MATH 143: Differential Geometry (Curves & Surfaces) SYLLABUS

**Course Description:** The course comprises applications of calculus and linear algebra to the geometry of curves and surfaces in space. Some gems along the way are: minimal surfaces, the Gauss-Bonnet Theorem (a beautiful theorem relating geometry and topology), and Stokes' Theorem (a theorem which encapsulates a number of theorems from multivariable calculus: Stokes' Theorem, Green's Theorem, and the Divergence Theorem).

As an upper division class in pure mathematics, there will be a reasonable number of proofs. However, the proofs should merely be a formalization of your visual thinking. There will also be more hands-on computations than typical upper division math classes.

**Official Course Description:** Geometry of curves and surfaces in three-space and higher dimensional manifolds. Parallel transport, curvature, and geodesics. Surfaces with constant curvature. Minimal surfaces.

## Teaching Staff:

**Instructor:** Dr. Laura Fredrickson  
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OFFICE HOURS: M 5-6pm, Tu 3-4pm

**Course Assistant:** Evangelie Zachos  
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OFFICE HOURS: W 1-3pm

**Lecture:** TuTh 1:30-2:50pm | Herrin-T185

**Exams:** There will be one midterm exam, taken in class. The final is comprehensive.

**Midterm:** Thursday, November 2 (in class)

**Final:** Monday, December 11 from 12:15-3:15pm 380-380X

**Prerequisites:** Math 53 or equivalent

**Course Text:** I will be following Neil Donaldson's *Differential Geometry* lecture notes. These are available from his website <https://www.math.uci.edu/~ndonalds/> under "Math 162A" and "Math 162B" and as a single document on Canvas. An optional complementary and classic text is DoCarmo's *Differential Geometry of Curves and Surfaces*. (Roughly, we'll cover Chapters 1-4.)

**Course website:** Course announcements, homework, and solutions will be posted on Canvas. Additionally, the syllabus will be posted on my website <http://web.stanford.edu/~ljfred4/>.

**Grading Policy:** On all work, your grade will be computed as a percentage: the number of points you earned divided by the number of points possible. The weekly homework and exams are weighted as follows:

- Homework: 25% (lowest score dropped)

- Midterm: 30%
- Final: 45%

Your letter grade will be given based on your numerical average earned in the class, on a scale not stricter than the following: you are guaranteed a D for 60.0 or above, C- for 70.0 or above, C for 73.0 or above, C+ for 77.0 or above, B- for 80.0 or above, B for 83.0 or above, B+ for 87.0 or above, A- for 90.0 or above, and an A for 93.0 or above.

**E-mail:** If you send me an e-mail, you can expect 24-hour turn-around on school days.

**Homework:** Weekly homework assignments are due at the beginning of class on Thursday. Alternatively, you can turn them in to the instructor's mailbox **by 1:20pm** on Thursday. The assignments will be posted on Canvas by the previous Wednesday.

The lowest score will be dropped to accommodate exceptional situations such as a serious illness. Because the lowest score is dropped, you can miss one assignment without penalty. No late homework will be accepted, and no make-up homework will be given.

You may hand write your solutions. Solutions should be readable and well-explained.

I encourage you to form study groups and work together. A good strategy is to try each problem yourself first, then get together with others to discuss your solutions and questions, and finally *you should write up the solutions by yourself*. (The Honor Code applies to this and all other written aspects of the course.)

**Alternate Sitting for the Midterm Exam:** In exceptional circumstances, and by *prearrangement only*, you may take the midterm exam at a fixed alternate time. The alternate sitting will always occur *before* the standard sitting for the exam. To arrange an alternate sitting you must e-mail me at least two weeks before the midterm.

**Final Exam Policy:** (See <http://registrar.stanford.edu/students/final-exams>.)

- Students must not register for classes with conflicting end-quarter exams.
- Alternative arrangements for the final may only be made for the following unforeseen circumstances: illness, personal emergency, or the student's required participation in special events (for example, athletic championships) approved as exceptions by the Committee on Undergraduate Standards and Policy (C-USP).

**Schedule:** This course is structured with the expectation that you will attend every lecture. Of course, sometimes an absence is necessary. In such a situation, you should contact a classmate to get notes and other information for the class you missed.

We will have 20 lectures in total. Here is a tentative schedule, which may be adjusted as the quarter goes on.

9/26 Parameterized curves, regular curves, arc-length parameterization

9/28 Frenet frame for a curve, curvature, torsion

10/3 Vector fields, tangent space, 1-forms, exterior derivative, line integrals

10/5 Parameterized surfaces, differential  $dx$ , regular implicitly defined surfaces

- 10/10 First and second fundamental forms, curves in surfaces, principal curvatures
- 10/12 Gauss and mean curvature, elliptic and hyperbolic points
- 10/17 Exterior calculus: wedge product, the exterior derivative
- 10/19 Moving frames for surfaces, structure equations
- 10/24 Adaptive frames
- 10/26 Gauss and mean curvatures, isometry, Gauss' Theorem Egregium
- 10/31 Riemannian geometry, black holes and relativity
- 11/2 **Midterm**
- 11/7 Geodesics
- 11/9 Parallel transport, covariant derivatives
- 11/14 Integration, Stokes' Theorem
- 11/16 Applications of Stokes' Theorem, Minimal surfaces
- 11/20-24 *Thanksgiving Recess*
- 11/28 Gauss-Bonnet Theorem
- 11/30 Gauss-Bonnet Theorem continued
- 12/5 TBD
- 12/7 Review

**Students with Documented Disabilities:** Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty dated in the current quarter in which the request is made. Students should contact the OAE *by the end of the first week of the quarter*, since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk (phone: 723-1066, <https://studentaffairs.stanford.edu/>).

**Course Text & Other Resources:** Neil Donaldson's notes are of high quality, and you should read them. DoCarmo's *Differential Geometry of Curves and Surfaces* provides a complementary perspective. Math texts are quite demanding on the reader, owing to the intrinsic difficulty of the material, so do not be surprised if you have to go slowly.

You are encouraged to attend the office hours provided by the instructor and course assistant.

Another resource which may be of use is Counseling and Psychological Services. See <http://vaden.stanford.edu/caps-and-wellness>.

**Computers:** If you wish to use a computer in class, you must speak with me first.

**Academic Integrity:** The Honor Code articulates Stanford University’s expectations of students and faculty in establishing and maintaining the highest standards in academic work. Examples of conduct that have been regarded as being in violation of the Honor Code (and are most relevant for this course) include copying from another’s examination paper or allowing another to copy from one’s own paper; plagiarism; revising and resubmitting a quiz or exam for regrading, without the instructor’s knowledge and consent; representing as one’s own work the work of another; and giving or receiving aid on an academic assignment under circumstances in which a reasonable person should have known that such aid was not permitted. See <http://communitystandards.stanford.edu/> for more information on the Honor Code.

**Important Dates:**

First Day of Classes .....	September 25, 2017
Add/Drop Deadline .....	October 13
Midterm Exam (in class) .....	November 2
Course Withdrawal & Change of Grading Basis Deadlines .....	November 17
Thanksgiving Recess (no classes) .....	November 20-24
Last Day of Classes, Last Day to Arrange an Incomplete .....	December 8
Final Exam .....	December 11, 12:15-3:15 p.m.