MATH 413/513: Introduction to Analysis I
SYLLABUS

Course Description: The three-quarter sequence Math 413-415 (513-515) gives a rigorous presentation of calculus of one and several variables (the material of Math 251-253 and Math 281-282). In Math 413/513, we will basic properties of $\mathbb{R}$ and $\mathbb{C}$, elementary point set topology and continuity in the context of metric spaces (which includes all subsets of $\mathbb{R}^n$ and $\mathbb{C}^n$), convergence of series, and differentiation.

Teaching Staff:

Instructor: Dr. Laura Fredrickson
E-mail: `lfredric@uoregon.edu`
Webpage: web.stanford.edu/∼ljfred4/
Office Hours: M 4:30-5:30, Tu 4:30-5:30 via Zoom (see Canvas for meeting code)

Lecture: MWF 12:30-1:20pm (Pacific Time)—Virtual (see Canvas for Zoom meeting info)

(Because of the COVID-19 academic disruption, Fall 2020 classes have been scheduled in either 60-minute or 90-minute blocks rather than in the standard blocks. The Math Department policy is to use the extra 10 minutes to provide breaks in order to make the Zoom meetings more bearable for students, and in this course we will be meeting 12:30-1:20pm.)

CRN: #14184 (Math 413); #14191 (Math 513)

Prerequisites: Math 282 (multi-variable calculus) and Math 317 (elementary analysis)

Course website: Course announcements, homework, solutions will be posted on Canvas.

Learning Outcomes: The course deals with:

- The basic topology of the real numbers and the complex numbers.
- The topology of metric spaces.
- Convergence of sequences and series of real numbers and complex numbers.
- Continuity of functions and continuous functions between metric spaces.
- Differentiability of functions, differentiable functions, and their derivatives, for functions with domain a suitable subset of the real numbers and codomain the real numbers or the complex numbers.

The successful student will be able to precisely state the definitions associated with these topics, and will be able to rigorously prove standard facts. The successful student will also be able to rigorously prove or disprove (as appropriate) statements about these topics which have not been encountered before, and which are at a level of difficulty appropriate to a mid-level introductory analysis course. The successful student will be able to write these proofs so that they are correct, complete, clear, readable, and in logical order.

Textbook: (Required) *Principles of Mathematical Analysis* by Walter Rudin, 3rd edition. (We will cover most of Chapters 1-5 in the fall quarter.)

I also recommend the following three references: Stephen Abbott's *Understanding Analysis* is well-written and and is easier than our textbook; it is one of the books used for the prerequisite sequence Math 281 & 282. Charles Pugh's *Real Mathematical Analysis* was written based on an honors real analysis course at UC Berkeley. Lastly, *Counterexamples in Analysis* by Gelbaum and Olmsted is a useful reference. If you're trying to understand why a hypothesis in a theorem is necessary, knowing a counterexample is illuminating. PDFs of some of these supplemental texts are available legally online through the University of Oregon library.

Additional Materials:

- You will need a green, red, and yellow object for in-class polling.
- You will need a webcam* so I can see your face during lecture.
- Make sure that you can scan homework to upload it to Canvas. There are a number of apps, e.g. Genius Scan, Turbo Scan, Cam Scanner (free) etc. You should scan in black and white rather than color, and upload your homework as a single PDF rather than a sequence of images.
- You will *not* need a printer.

Let me know if any of these will be a problem for you. (See here.)

What to Expect in this Remote Class: This will be an interactive class. I'll be using the "breakout rooms" feature in Canvas where you can discuss a question with 2-4 of your classmates. You'll be able to see both my face and the notes, and I ask that you turn on your video as well. (Your facial expressions are an important source of feedback for me as I'm lecturing, and I need them! It helps me know when to slow down or clarify something.) I'll also be doing some visual polling. For this, *you will need a solid green, yellow, and red piece of paper.* Have these nearby for every lecture, and raise one of them to respond when prompted. I'll use these for multiple choice questions, and for qualitative questions like "How are you all doing with this new material?" If it makes sense raise green, if you're confused raise red, and if you're somewhere in the middle raise yellow. Your responses are another essential form of feedback for me. (See further advice here.)

The lectures will be recorded and posted to Canvas; a PDF copy of the notes will also be posted to Canvas.

Course Etiquette for Lectures: *(This may be refined as the course progresses.)* When your camera is on, please be mindful of your surroundings and try to place yourself somewhere that won't have distracting objects or activity in the background. Please keep your microphone muted when you are not speaking. If you have a question, please type it in the chat box. Wait until the instructor calls on you to speak.

Grading Policy: On all work, your grade will be computed as a percentage: the number of points you earned divided by the number of points possible. The weekly homework and exams are weighted as follows:

- Homework: 25% (lowest two scores dropped)
- Midterm #1: 20%
- Midterm #2: 20%
- Final: 25%
- Participation: 10%

Your letter grade will be given based on your numerical average earned in the class, on a scale not stricter than the following: you are guaranteed a D for 60.0 or above, C- for 70.0 or above, C for 73.0 or above, C+ for 77.0 or above, B- for 80.0 or above, B for 83.0 or above, B+ for 87.0 or above, A- for 90.0 or above, and an A for 93.0 or above.

Homework: Weekly homework assignments are to be submitted via **Canvas** by 1:30pm each Wednesday. Your scans should be of high quality, easily readable, correctly oriented, and in the correct order. The assignments will be posted on Canvas by the previous Wednesday. For ease of online grading, *each problem should be on its own sheet(s) of paper*. Each assignment will be out of 50 points, and only a subset of the problems will be graded weekly.

The lowest two scores will be dropped to accommodate exceptional situations such as a serious illness or ordinary mishaps like forgetting to turn in the completed assignment. Because the lowest two score is dropped, you can miss two assignments without penalty.

Your solutions should be clear and well-explained. E.g., you should try to use complete sentences, insert explanations, and err on the side of writing out "for all," etc. rather than using the symbol. Professor Keith Conrad at the University of Connecticut has written a helpful guide to common errors in mathematical writing: http://www.math.uconn.edu/~kconrad/math216/mathwriting.pdf. (While you're welcome to type up your solutions using LaTeX, I personally wouldn't recommend it since it's more time-consuming.)

I encourage you to form study groups and work together. A good strategy is to try each problem yourself first, then get together with others to discuss your solutions and questions, and finally *you should write up the solutions by yourself*. Writing up your solutions by yourself will keep your good collaboration from veering into homework copying.

In this course, homeworks are graded by a graduate employee (GE). The homework grader is in a later stage of the program from the GEs enrolled in the course, and so this should minimize the chances of any conflict of interest occurring with one GE grading the work of another. However, sometimes unexpected conflicts do occur. If at any point during the quarter a graduate student enrolled in this course has a concern related to a conflict of interest, privacy, fairness, or any other aspects of the graders conduct please come talk to me about it. You may also speak directly with the Director of Graduate Studies or Department Head about your concerns. If such concerns arise, you may request that I grade your homework instead of the GE grader.

Exams: There will be two midterm exams and one final exam. The midterm exams will be released at 1:30pm on a Monday and due 24-hours later. The exams will be written to be shorter than 90-minutes so the additional time is for your scheduling flexibility.

- Midterm #1 released Monday, October 26 at 1:30pm; due Tuesday, October 27 at 1:30pm.
- Midterm #2 released Monday, November 16 at 1:30pm; due Tuesday, November 17 at 1:30pm.
- The final exam will be given during assigned time slot.

E-mail: If you send me an e-mail, you can expect 24-hour turn-around on school days.

Participation: I've added a participation grade to help you succeed and stay engaged in this online environment. The 10% is broken down as follows:

- Attendance in Lecture (6%): There are 28 lectures, and you may miss four classes without penalty. To get credit, *you must also have your video on and you must enter your full name as your screen name*. (Your facial expressions are an important source of feedback for me as I'm lecturing, and I need them! It helps me know when to slow down or clarify something.

Your names will help me connect names with faces.) If for some reason you cannot have your video on during a given lecture, you must ask or answer two questions in the chat.

- Presentation of a Problem (3%): During the quarter, I will meet with each of you one-on-one for 10 minutes. You'll present some pre-assigned homework problem to me. This is an opportunity for you to practice expressing yourself clearly in mathematics.
- Student Questionnaire (1%): In the first week of class, you'll be asked to fill out a Student Questionnaire to help me get to know you better, and facilitate some course logistics. This is worth 1%.

If you're in a time zone where the lecture is the middle of the night, we will obviously change the computation of the participation grade.

Office Hours: Virtual office hours are times you can meet with your instructor to discuss the material being covered in class, questions or concerns you might have, and other related issues. Join the Zoom meeting during office hour times using the links provided in Canvas. Feel free to join even if you don't have any questions. You can listen in on the conversation, which might spark a question for you!

Notes on Textbook: The primary textbook is *Principles of Mathematical Analysis,* by Walter Rudin. The textbook is of high quality, and you should read it. This does not mean that it is "easy" to read. Math books are quite demanding on the reader, owing to the intrinsic difficulty of the material, so do not be surprised if you have to go slowly.

The book contains a few oddities of notation and terminology which I will not follow. Open intervals are referred to as "segments". The sets of real numbers and rational numbers are called R and Q instead of the more usual $\mathbb{R}$ and $\mathbb{Q}$. (In particular, I will feel free to use the letters R and Q for other purposes.) In contrast to this, the book suddenly switches from f to $\mathbf{f}$ when the function is supposed to take values in $\mathbb{R}^n$ instead of a general metric space; I will make no such distinction.

To avoid confusion about whether, say, an increasing sequence $(x_n)_{n\in\mathbb{N}}$ in $\mathbb{R}$ is supposed to satisfy $x_{n+1} > x_n$ or $x_{n+1} \geq x_n$, in anything formal I will refer only to "strictly increasing" and "nondecreasing" respectively sequences (as well as functions, etc.). Similarly, I will use "strictly decreasing" and "nonincreasing." To see the problem, compare Rudin's definition of a monotonically increasing sequence with the definition of an increasing sequence in the textbook we are currently using in Math 251–253. For similar reasons, I will avoid "positive" in favor of "nonnegative" and "strictly positive".

Rudin (following a number of authors) defines a set E to be "countable" if there is a *bijection* from E to $\mathbb{N}$. More typically, other others define a set E to be "countable" if there is a *surjection* from E to $\mathbb{N}$; thus finite sets are considered countable. Rudin's choice is poor: many of the authors (including Rudin) who exclude finite sets in their definition of "countable" nevertheless implicitly include finite sets in practice. One sees this, for example, in Rudin's Problem 2.22. Everybody agrees that a metric space with only one point is separable. But, formally, Rudin's definition excludes such spaces: they don't have countable subsets because they don't have infinite subsets. (A careful author would have written "at most countable".) The wording "a metric space is separable if it has a countable dense subset" (and in similar related definitions) is nearly universal. Therefore I will take "countable" to include finite subsets.

Mental Health and Wellness: Life at college and in graduate school can be very complicated, especially in these times. Students often feel overwhelmed or stressed, experience anxiety or depression, struggle with relationships, or just need help navigating challenges in their life. If you're facing such challenges, you don't need to handle them on your own—there's help and support on campus.

As your instructor if I believe you may need additional support, I will express my concerns, the reasons for them, and refer you to resources that might be helpful. It is not my intention to know the details of what might be bothering you, but simply to let you know I care and that help is available. Getting help is a courageous thing to do—for yourself and those you care about.

University Health Services help students cope with difficult emotions and life stressors. If you need general resources on coping with stress or want to talk with another student who has been in the same place as you, visit the Duck Nest (located in the EMU on the ground floor) and get help from one of the specially trained Peer Wellness Advocates. Find out more at `health.uoregon.edu/ducknest`.

University Counseling Services (UCS) has a team of dedicated staff members to support you with your concerns, many of whom can provide identity-based support. All clinical services are free and confidential. Find out more at counseling.uoregon.edu or by calling 541-346-3227 (anytime UCS is closed, the After-Hours Support and Crisis Line is available by calling this same number).

Accessible Education: The University of Oregon is working to create inclusive learning environments. Please notify me if there are aspects of the instruction or design of this course that result in disability-related barriers to your participation. You are also encouraged to contact the Accessible Education Center in 360 Oregon Hall at 541-346-1155 or uoaec@uoregon.edu. For more information, see `https://aec.uoregon.edu/`.

Eduational Equity: There are rightly a lot of discussions about educational inequity, whether that inequity is caused by living situation, time zone, disabilities, etc. These are real concerns. This syllabus is written from the perspective that, in general, it's far better to make a policy and proceed by it, but reconsider aspects when obviously necessarily. E.g. of course we'll change the way that participation is computed if you're currently in Asia or Australia and the lectures are happening in the middle of the night for you. Similarly, if you will not be attending the lectures in a place you feel comfortable turning on your webcam, we will find an alternate solution. We write out the general policy because it provides transparency, clarity, and a sense of normalcy since all learning environments have guide rails. Please send me an e-mail if you have a problem. I care about you. And if you're concerned about educational equity for others, know that so are we, and we are working to mitigate it.

Academic Integrity: The University Student Conduct Code (available at `conduct.uoregon.edu`) defines academic misconduct. Students are prohibited from committing or attempting to commit any act that constitutes academic misconduct. By way of example, students should not give or receive (or attempt to give or receive) unauthorized help on assignments or examinations without express permission from the instructor. If there is any question about whether an act constitutes academic misconduct, it is the students obligation to clarify the question with the instructor before committing or attempting to commit the act.

Course Privacy Statement: Students may not audio or video record class meetings without permission from the instructor. For videos posted by the instructor, students may keep these recordings only for personal use and may not post recordings on the internet or otherwise distribute them. These policies protect the privacy rights of instructors and students, and the intellectual property and other rights of the university.

It is important that you do not share our course Zoom links or meeting passwords with anyone outside of our course to protect the privacy of everyone in attendance.

Advice: I strongly encourage you to attend lecture. While the lectures will be recorded and posted, and the notes are being posted as well, I think you'll find that recorded live lectures with no editing are painful to watch. (I've watched enough to know this personally.) I would suspect that despite your best intention of watching the recorded lectures, you'll find yourself putting off watching the lectures, or choosing to watch Netflix instead. Additionally, the purpose of lectures is not just for you to absorb material, but to contribute. You'll get a lot more out of this class if you take this perspective.

Important Dates:

First day of classes	September 29, 2020
Last day to drop classes without 'W'	October 3
Last day to add classes	October 5
Midterm #1 due	October 27
Last day to withdraw from classes	November 15
Midterm #2 due	November 17
Thanksgiving vacation	November 26-27
Last day of classes	December 4
Final examination period	December 7-11
Final exam	TBD