MATH 53: Ordinary Differential Equations with Linear Algebra
SYLLABUS

Course Description: This is a first course in Ordinary Differential Equations. The material includes: separation of variables; integrating factors and exact differential equations; systems of linear differential equations; eigenvalues and eigenvectors; the method of variation of parameters; the Laplace transform; systems of nonlinear differential equations; conservation laws.

Teaching Staff:

> **Instructor:** Dr. Laura Fredrickson
> E-MAIL: `lfredrickson@stanford.edu`
> WEBPAGE: `web.stanford.edu/~ljfred4/`
> OFFICE: 380-382L
> OFFICE HOURS: Monday 11:25-1:25pm

> **TA:** Jesse Madnick
> E-MAIL: `jmadnick@math.stanford.edu`
> OFFICE: 380-380T
> OFFICE HOURS: Tuesday 1:40-2:40pm, Wednesday 4-5pm

> **TA:** Jesse Silliman
> E-MAIL: `silliman@stanford.edu`
> OFFICE: 380-384M
> OFFICE HOURS: Tuesday 5-6pm, Wednesday 5-6pm

Lectures & Sessions:

> **Lecture-01** MWF 9:30-10:20am | 380-380X | Class # 9567 (Laura Fredrickson)
> **Lecture-02** MWF 10:30-11:20am | 200-203 | Class # 9659 (Laura Fredrickson)

> **TA Session-03** TuTh 9:30-10:20am | Green Earth Sciences 131 (Jesse Silliman)
> **TA Session-04** TuTh 10:30-11:20am | 320-220 (Jesse Silliman)
> **TA Session-05** TuTh 11:30-12:20am | Gates B12 (Jesse Madnick)
> **TA Session-06** TuTh 12:30-1:20pm | Green Earth Sciences 150 (Jesse Madnick)

Exams: There will be two midterm exams, taken in the evening. The final is comprehensive.

> **Midterm #1:** Thursday, October 20 from 7-10 pm in Bishop Auditorium
> **Midterm #2:** Thursday, November 10 from 7-10 pm in Bishop Auditorium
> **Final:** Monday, December 12 from 7-10pm (Location TBA)

Prerequisites: Single Variable Calculus (Math 19-20-21, or 41-42, or equivalent); Linear Algebra and Multivariable Calculus (Mathematics 51 or equivalent)

Textbook: (Required*) *Differential Equations with Boundary Value Problems*, 2nd edition, by James Brannan and William Boyce (ISBN: 978-0-470-59535-0)

***Affordability of Course Materials:** Stanford University and its instructors are committed to ensuring that all courses are financially accessible to all students. If you are an undergraduate who needs assistance with the cost of course textbooks, supplies, materials and/or fees, you are welcome to approach me directly. If would prefer not

Course website: Course announcements, homework, solutions will be posted on Canvas. Additionally, the course website is `web.stanford.edu/class/math53/`.

Grading Policy: On all work, your grade will be computed as a percentage: the number of points you earned divided by the number of points possible. The weekly homework and exams are weighted as follows:
- Homework: 20% (lowest score dropped)
- Midterm 1: 20%
- Midterm 2: 20%
- Final: 40%

Your letter grade will be given based on your numerical average earned in the class, on a scale not stricter than the following: you are guaranteed a D for 60 or above, C- for 70 or above, C for 73 or above, C+ for 77 or above, B- for 80 or above, B for 83 or above, B+ for 87 or above, A- for 90 or above, and an A for 93 or above.

Homework: The only way to learn mathematics is to do mathematics! I encourage you to form study groups and work together. A good strategy is to try each problem yourself first, then get together with others to discuss your solutions and questions, and finally *write up the solutions yourself.* Please work out problems neatly—don't hand in your scratch work. One course goal is to sharpen your mathematical writing skills, and homework is a place to practice.

Weekly homework assignments are to be turned in to your TA at *the beginning* of your Thursday discussion section. The assignments will be posted on Canvas the previous Wednesday.

The lowest score will be dropped to accommodate exceptional situations such as a serious illness. Because the lowest score is dropped, you can miss one assignment without penalty. No late homework will be accepted, and no make-up homework will be given.

Alternate Sitting for Midterm Exams: In exceptional circumstances, and by *prearrangement only*, you may take a midterm exam at a fixed alternate time. The alternate sitting will always occur *before* the standard sitting for the exam. To arrange an alternate sitting you must e-mail me by October 10 for Midterm #1 or by October 31 for Midterm #2.

Final Exam Policy: (See `http://registrar.stanford.edu/students/final-exams`.)
- Students must not register for classes with conflicting end-quarter exams.
- Alternative arrangements for the final may only be made for the following unforeseen circumstances: illness, personal emergency, or the student's required participation in special events (for example, athletic championships) approved as exceptions by the Committee on Undergraduate Standards and Policy (C-USP).

Schedule: This course is structured with the expectation that you will attend every lecture and discussion session. Of course, sometimes an absence is necessary. In such a situation, you should contact a classmate to get notes and other information for the class you missed.

We will have 30 lectures in total. Here is a tentative schedule, which may be adjusted as the quarter goes on.

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to approach me directly, please note that you can ask the Diversity & First-Gen Office for assistance by completing their questionnaire on course textbooks & supplies: http://tinyurl.com/jpqbarn or by contacting Joseph Brown, the Associate Director of the Diversity and First-Gen Office (jlbrown@stanford.edu; Old Union Room 207). Dr. Brown is available to connect you with resources and support while ensuring your privacy.

WEEK 1 (9/26-9/30):
- Direction fields; equilibrium solutions; qualitative behavior (§1.1)
- First order linear differential equations with variable coefficients; integrating factors (§1.2)
- Review of complex numbers (Appendix B)

WEEK 2 (10/3-10/7):
- Eigenvalues and eigenvector for matrices (Appendix A.4)
- Similar matrices and matrix exponentials (§6.5)

WEEK 3 (10/10-10/14):
- Systems of linear differential equations (§3.2)
- Solving homogeneous systems with constant coefficients (§3.3)
- Fundamental systems and Wronskian determinant (§3.3)
- Matrices with complex eigenvalues (§3.4)

WEEK 4 (10/17-10/21) + Midterm #1:
- Matrices with repeated eigenvalues; Phase space plots; asymptotic behavior of solutions (§3.5)
- Variation of parameters (only constant matrix case) (§4.7)
- The method of separation of variables; examples of separable differential equations (§2.1)

WEEK 5 (10/24-10/28):
- Existence and uniqueness theory for ordinary differential equations, the interval of existence of a solution; analysis of equilibria (§2.3)
- The logistic growth model; autonomous differential equations (§2.4)
- Euler's method as "connecting the dots" of a direction field (§1.3)
- Improved Euler and Runge-Kutta methods (§2.7)

WEEK 6 (10/31-11/4):
- Existence and uniqueness theory for systems of linear ODE (§3.2)
- Rewriting a differential equation of second order as a coupled system of two differential equations of first order; matrix notation (§3.2)
- Linear homogeneous second degree equations (§4.3)
- Second order linear differential equations; damped linear oscillations in mechanics (§4.4)

WEEK 7 (11/7-11/11) + Midterm #2:
- Non-homogeneous equations: method of undetermined coefficients (§4.5)
- Definition and properties of Laplace transform (§5.1, 5.2)
- Inverse Laplace transform (§5.3)

WEEK 8 (11/14-11/18): *(followed by Thanksgiving Break)*
- Solving differential equation with Laplace transform (§5.4)
- Review of power series (§8.1)
- Series solution to Airy equation (§8.2)

WEEK 9 (11/28-12/2):
- Fourier series (§9.2)
- Fourier series for simple functions (§9.3)
- Laplace equation for a rectangle (§9.8)

WEEK 10 (12/5-12/9):
- Review

Students with Documented Disabilities: Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty dated in the current quarter in which the request is made. Students should contact the OAE *by the end of the first week of the quarter*, since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk (phone: 723-1066, URL: studentaffairs.stanford.edu).

Textbook and other Resources: The primary textbook is *Differential Equations with Boundary Value Problems* by Brannan and Boyce. The textbook is of high quality, and you should read it. This does not mean that it is "easy" to read. Mathe books are quite demanding on the reader, owing to the intrinsic difficulty of the material, so do not be surprised if you have to go slowly.

You are encouraged to attend the office hours provided by the instructor and teaching assistants. You may attend the office hours of either teaching assistant.

The Stanford University Mathematics Organization (SUMO) offers free drop-in tutoring for students in the 50's sequence. Tutoring is available on Mondays and Wednesdays from 6-10pm in room 380-381T, and on Tuesdays from 7-9 pm in room 380-381U. The Center for Teaching & Learning also provides free tutoring.

Another resource which may be of use is Counseling and Psychological Services. See `http://vaden.stanford.edu/caps-and-wellness`.

Computers: If you wish to use a computer in class, you must speak with me first.

Academic Integrity: The Honor Code articulates Stanford University's expectations of students and faculty in establishing and maintaining the highest standards in academic work. Examples of conduct that have been regarded as being in violation of the Honor Code (and are most relevant for this course) include copying from another's examination paper or allowing another to copy from one's own paper; plagiarism; revising and resubmitting a quiz or exam for regrading, without the instructor's knowledge and consent; representing as one's own work the work of another; and giving or receiving aid on an academic assignment under circumstances in which a reasonable person should have known that such aid was not permitted. See `http://communitystandards.stanford.edu/` for more information on the Honor Code.

Important Dates:

Add/Drop Deadline .... October 14
Midterm #1 .... October 20
Midterm #2 .... November 10
Final Exam .... December 12