This document contains further mild corrections and explanations for \[\text{[Lip06]},\] beyond those corrected in \[\text{[Lip14]}.\]

Page 26 of \[\text{[Lip14]}.\] In the proof of Proposition 4.2', the instances to \(\pi_\mathcal{D} \circ u\) should be \(\pi_\Sigma \circ u\).

(Thanks to Morgan Weiler for pointing out this typo.)

Page 1001 of \[\text{[Lip06]}.\] In Proposition 8.6, the proof that \(A_{\zeta}\) induces an action of the exterior algebra is incorrect: the moduli spaces \(\hat{M}^A_{K,2}\) have an unaccounted for end where \(p_1 \to p_2\). To correct the proof, we first show that for any \(\zeta, \eta \in H_1(Y)\), the map \(A_\zeta \circ A_\eta + A_\eta \circ A_\zeta = 0\) on \(\hat{HF}, \hat{HF}^+, \hat{HF}^-,\) and \(\hat{HF}^\infty\). To see this, choose disjoint knots \(K_\zeta, K_\eta \subset \Sigma \times [0,1]\) representing \(\zeta\) and \(\eta\), and consider the index 2 moduli space of holomorphic curves with one point mapped to \(\zeta\) and a second point mapped to \(\eta\). The ends of this moduli space show that \(A_\zeta \circ A_\eta + A_\eta \circ A_\zeta\) is chain homotopic to the zero map. Next, to see that \(A^2_\zeta = 0\) on Floer homology it suffices to consider the case that \(\zeta\) is represented by a chain \(K\) in \(\Sigma\) which is dual to some \(\alpha_i\), i.e., \(K\) intersects \(\alpha_i\) in one point and is disjoint from \(\alpha_j\) for \(j \neq i\). Let \(K'\) be a small isotopic translate of \(K\), and consider the moduli space of holomorphic curves

\[\{u: (S, p, q) \rightarrow \Sigma \times [0,1] \times \mathbb{R} | \pi_\Sigma(u(p)) \in K, \pi_\Sigma(u(q)) \in K', \pi_\mathbb{R}(u(p)) - \pi_\mathbb{R}(u(q)) > 0\}\]

(and with \(u\) satisfying the conditions \((M0)-(M6)\) from the paper). This moduli space has no end with \(\pi_\mathbb{R}(u(p)) - \pi_\mathbb{R}(u(q)) \to 0\) because \(K\) and \(K'\) are disjoint and intersect the \(\alpha\)-circles in a single point. Then, it is easy to see that the ends of the moduli space imply that \(A^2_\zeta\) is chain homotopic to 0.

(Thanks to Ian Zemke for pointing out this mistake.)

Page 1005 of \[\text{[Lip06]}.\] In the proof of Lemma 9.3, the fact that the ends of \(\hat{M}_1(\vec{x}, \vec{y}, k)\) correspond to height 2 holomorphic buildings in which the \(\mathbb{R}\)-invariant level has ind = 1 and the non-\(\mathbb{R}\)-invariant level has ind = 0 is not sufficiently justified, because Proposition 4.2 was only proved with respect to \(\mathbb{R}\)-invariant almost complex structures. The easiest solution is to define \(\Phi\) to only count embedded, rigid holomorphic curves in homology classes with ind = 0. (This is, in some sense, three conditions: the combinatorial index \(\text{ind}(A) = c(A) + n_x(A) + n_\mathbb{R}(A) = 0\), the curve must be embedded, and the curve must lie in a 0-dimensional moduli space. Presumably the condition that \(\text{ind}(A) = 0\) implies the other two, but that has not been shown for non-\(\mathbb{R}\)-invariant almost complex structures.) Similarly, define \(\hat{M}_1(\vec{x}, \vec{y}, k)\) to consist of ind\((A) = 1, 1\)-dimensional moduli spaces of embedded curves with \(n_3 = k\). Since ind\((A)\) agrees with the dimension of the moduli space of curves for \(\mathbb{R}\)-invariant levels

\(\text{[Lip14]}\)
and is additive under gluing, if a sequence of curves in $\mathcal{M}_1(\vec{x}^1, \vec{y}^2, k)$ converges to a 2-
story holomorphic building then the $\mathbb{R}$-invariant level must have $\text{ind}(A) = 1$, so the non-$\mathbb{R}$-
invariant level must have $\text{ind}(A) = 0$. Note also that gluing preserves embeddedness and
non-embeddedness. It follows that the ends of $\bigcup_k \mathcal{M}_1(\vec{x}^1, \vec{y}^2, k)$ correspond to the terms in
$\partial \circ \Phi + \Phi \circ \partial$, as desired.

(Thanks to Cagatay Kutluhan for pointing out this gap.)

Page 1022 of [Lip06]. In the formula defining $K$, the last instance of $B_{\alpha,\gamma}$ should be
$B_{\alpha,\beta}$.

(Thanks to Michael Gartner for pointing out this mistake.)

Page 1043 of [Lip06]. The explicit description of the Hamiltonian $H$ is incorrect. Figure
14 is correct, and clearer. Finding a correct formula for such a Hamiltonian is an easy exercise
(which the author solved incorrectly). An explanation of why the resulting cylinders are La-
grangian with respect to a suitable symplectic form is also missing; see, for instance, [LOT14,
Formula (3.25)]. (This is also relevant to the proof of isotopy invariance in Section 9.)

The labels $w^+_i$ and $w^-_i$ in Figure 15 also seem to be reversed (if the author still understands
the notation), making the proof of Lemma 11.8 unconvincing. A more convincing proof is
the fact that the 1-gon maps are homogeneous with respect to the Maslov grading and, by
any model computation (e.g., in genus 1) the top-graded generator is an output of the 1-gon
maps. See the proof of Proposition 11.4.

(Thanks to Thomas Hockenhull for pointing out these mistakes.)

Page 1051 of [Lip06]. In the enumerated list at the bottom of the page, point (2) says
that the $S'_i$ are closed surfaces, which contradicts point (5). The $S'_i$ should be the union of
a closed surface with a bigon.

(Thanks to Michael Gartner for pointing out this mistake.)

REFERENCES


[Lip14] Robert Lipshitz, Correction to the article: A cylindrical reformulation of Heegaard Floer homology

[LOT14] Robert Lipshitz, Peter S. Ozsváth, and Dylan P. Thurston, Bordered Floer homology and the
spectral sequence of a branched double cover II: the spectral sequences agree, J. Topol. 9 (2014),