This document contains further mild corrections and explanations for [Lip06], beyond those corrected in [Lip14].

Page 26 of [Lip14]. In the proof of Proposition 4.2', the instances to $\pi_D \circ u$ should be $\pi_\Sigma \circ u$.

(Thanks to Morgan Weiler for pointing out this typo.)

Page 1001 of [Lip06]. In Proposition 8.6, the proof that $A_\zeta$ induces an action of the exterior algebra is incorrect: the moduli spaces $\hat{\mathcal{M}}^A_{K,2}$ have an unaccounted for end where $p_1 \to p_2$. To correct the proof, we first show that for any $\zeta, \eta \in H_1(Y)$, the map $A_\zeta \circ A_\eta + A_\eta \circ A_\zeta = 0$ on $\hat{HF}, \hat{HF}^+, \hat{HF}^-, \text{and } \hat{HF}^\infty$. To see this, choose disjoint knots $K_\zeta, K_\eta \subset \Sigma \times [0,1]$ representing $\zeta$ and $\eta$, and consider the index 2 moduli space of holomorphic curves with one point mapped to $\zeta$ and a second point mapped to $\eta$. The ends of this moduli space show that $A_\zeta \circ A_\eta + A_\eta \circ A_\zeta$ is chain homotopic to the zero map. Next, to see that $A^2_\zeta = 0$ on Floer homology it suffices to consider the case that $\zeta$ is represented by a chain $K$ in $\Sigma$ which is dual to some $\alpha_i$, i.e., $K$ intersects $\alpha_i$ in one point and is disjoint from $\alpha_j$ for $j \neq i$. Let $K'$ be a small isotopic translate of $K$, and consider the moduli space of holomorphic curves

$$\{u: (S,p,q) \to \Sigma \times [0,1] \times \mathbb{R} \mid \pi_\Sigma(u(p)) \in K, \pi_\Sigma(u(q)) \in K', \pi_\mathbb{R}(u(p)) - \pi_\mathbb{R}(u(q)) > 0\}$$

(and with $u$ satisfying the conditions (M0)–(M6) from the paper). This moduli space has no end with $\pi_\mathbb{R}(u(p)) - \pi_\mathbb{R}(u(q)) \to 0$ because $K$ and $K'$ are disjoint and intersect the $\alpha$-circles in a single point. Then, it is easy to see that the ends of the moduli space imply that $A^2_\zeta$ is chain homotopic to 0.

(Thanks to Ian Zemke for pointing out this mistake.)

Page 1005 of [Lip06]. In the proof of Lemma 9.3, the fact that the ends of $\hat{\mathcal{M}}_1(x^1, y^2, k)$ correspond to height 2 holomorphic buildings in which the $\mathbb{R}$-invariant level has ind = 1 and the non-$\mathbb{R}$-invariant level has ind = 0 is not sufficiently justified, because Proposition 4.2 was only proved with respect to $\mathbb{R}$-invariant almost complex structures. The easiest solution is to define $\Phi$ to only count embedded, rigid holomorphic curves in homology classes with ind = 0. (This is, in some sense, three conditions: the combinatorial index $\text{ind}(A) = e(A) + n_\mathbb{R}(A) + n_\mathbb{C}(A) = 0$, the curve must be embedded, and the curve must lie in a 0-dimensional moduli space. Presumably the condition that $\text{ind}(A) = 0$ implies the other two, but that has not been shown for non-$\mathbb{R}$-invariant almost complex structures.) Similarly, define $\hat{\mathcal{M}}_1(x^1, y^2, k)$ to consist of $\text{ind}(A) = 1$, 1-dimensional moduli spaces of embedded curves with $n_3 = k$. Since $\text{ind}(A)$ agrees with the dimension of the moduli space of curves for $\mathbb{R}$-invariant levels

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and is additive under gluing, if a sequence of curves in $\overline{\mathcal{M}}_1(\vec{x}, \vec{y}, k)$ converges to a 2-story holomorphic building then the $\mathbb{R}$-invariant level must have ind$(A) = 1$, so the non-$\mathbb{R}$-invariant level must have ind$(A) = 0$. Note also that gluing preserves embeddedness and non-embeddedness. It follows that the ends of $\bigcup_k \overline{\mathcal{M}}_1(\vec{x}, \vec{y}, k)$ correspond to the terms in $\partial \circ \Phi + \Phi \circ \partial$, as desired.

(Thanks to Cagatay Kutluhan for pointing out this gap.)

Page 1043 of [Lip06]. The explicit description of the Hamiltonian $H$ is incorrect. Figure 14 is correct, and clearer. Finding a correct formula for such a Hamiltonian is an easy exercise (which the author solved incorrectly). An explanation of why the resulting cylinders are Lagrangian with respect to a suitable symplectic form is also missing; see, for instance, [LOT14, Formula (3.25)]. (This is also relevant to the proof of isotopy invariance in Section 9.)

The labels $w^+_i$ and $w^-_i$ in Figure 15 also seem to be reversed (if the author still understands the notation), making the proof of Lemma 11.8 unconvincing. A more convincing proof is the fact that the 1-gon maps are homogeneous with respect to the Maslov grading and, by any model computation (e.g., in genus 1) the top-graded generator is an output of the 1-gon maps. See the proof of Proposition 11.4.

(Thanks to Thomas Hockenhull for pointing out these mistakes.)

References


[Lip14], Correction to the article: A cylindrical reformulation of Heegaard Floer homology [mr2240908], Geom. Topol. 18 (2014), no. 1, 17–30.