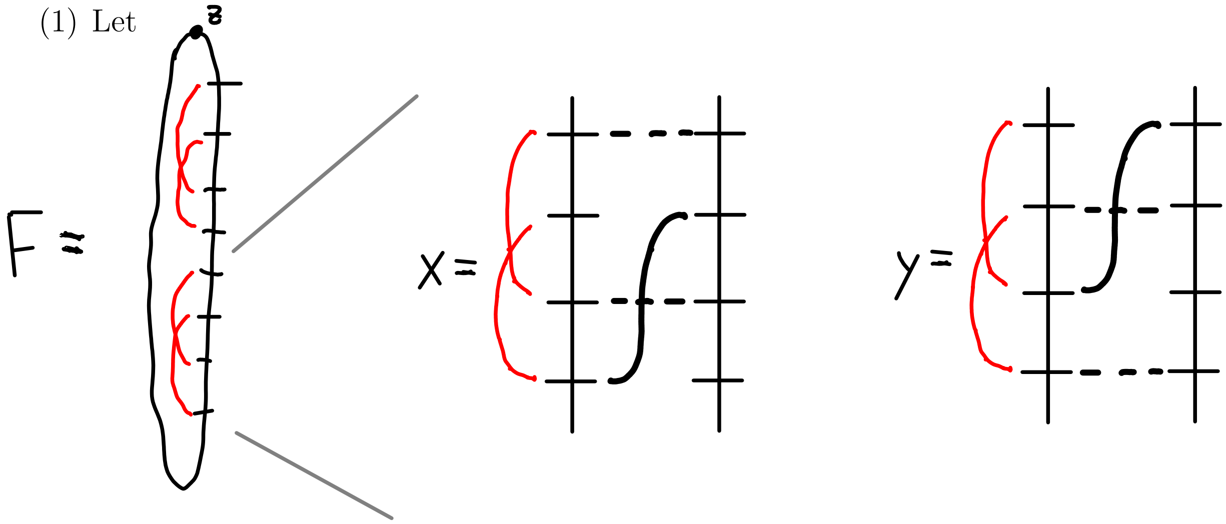
BORDERED FLOER HOMOLOGY HOMEWORK 1

ROBERT LIPSHITZ

Problems roughly in order of increasing difficulty. Solutions available on request; (partial) solutions can also be found in the references. (None are intended to be terribly hard.)

(1) Let



Verify that $d(x) \cdot y = y \cdot x$. Conclude that for F the split genus 2 matching shown on the left above, the differential algebra $\mathcal{A}(F)$ has no $\mathbb{Z}$-grading.

(2) Give a completely algebraic description of $\mathcal{A}(n, k)$. (Available in [1, Section 3.1.1].)

(3) Compute the homology of $\mathcal{A}(n, k)$. (Available in [2, Section 4], along with a computation of the additive structure of $H_*(\mathcal{A}(\mathcal{Z}))$.)

(4) Let F be a closed, orientable surface. Fix a nonvanishing section v_0 of $TF \oplus \mathbb{R}$. Let $\mathcal{V}$ denote the set of vector fields v on $[0,1] \times F$ such that $v|_{0\times F} = v|_{1\times F} = v_0$. The set $\mathcal{V}$ is a group under concatenation in the $[0,1]$-factor. Prove that this group G is a $\mathbb{Z}$-central extension of $H_1(F)$.

(5) Fix a complex structure j on a surface Σ. The induced complex structure $j^{\times g}$ on $\Sigma^{\times g}$ descends to a complex structure $\mathrm{Sym}^g(j)$ on $\mathrm{Sym}^g(\Sigma)$. Prove: with respect to the complex structure $\mathrm{Sym}^g(j)$, there is a canonical bijection between holomorphic maps $u\colon \mathbb{D}^2 \to \mathrm{Sym}^g(\Sigma)$ (not contained in the diagonal) and diagrams

$$\begin{tikzcd} S \arrow[r, "u_\Sigma"] \arrow[d, "u_{\mathbb{D}}"'] & \Sigma \\ \mathbb{D}^2 & \end{tikzcd}$$

Date: July 19, 2010.

where S is a Riemann surface, $u_{\mathbb{D}}$ and u_{Σ} are holomorphic and $u_{\mathbb{D}}$ is a g-fold branched cover. (Smith [4] calls this the *tautological correspondence*. This fact will be used extensively in Lecture 2. See for instance [3, Lemma 3.6] for help.)

(6) Verify that, in general, $\mathcal{A}(\mathcal{Z})$ is not formal.

References

[1] Robert Lipshitz, Peter S. Ozsváth, and Dylan P. Thurston, *Bordered Heegaard Floer homology: Invariance and pairing*, 2008, arXiv:0810.0687.

[2] ———, *Bimodules in bordered Heegaard Floer homology*, 2010, arXiv:1003.0598.

[3] Peter S. Ozsváth and Zoltán Szabó, *Holomorphic disks and topological invariants for closed three-manifolds*, Ann. of Math. (2) **159** (2004), no. 3, 1027–1158, arXiv:math.SG/0101206.

[4] Ivan Smith, *Serre-Taubes duality for pseudoholomorphic curves*, Topology **42** (2003), no. 5, 931–979, arXiv:math.SG/0106220.

Department of Mathematics, Columbia University, New York, NY 10027
E-mail address: `lipshitz@math.columbia.edu`