MATH 431/531 FALL 2016
HOMEWORK 4
DUE OCTOBER 19, 2016.

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Problems:

(1) Lee, Exercise 2.13.
(2) Let X be a set and $\mathcal{U}$ the indiscrete topology in X. When does a sequence $(x_i)_{i=1}^{\infty}$ in X converge to a point $x \in X$?
(3) Continuity can *not* be rephrased as saying that the image of every open set is open. To see this:
  (a) Give an example of topological spaces X and Y, a continuous map $f\colon X \to Y$, and an open set $U \subset X$ so that $f(U)$ is not open in Y.
  (b) Give an example of topological spaces X and Y, a map $f\colon X \to Y$ so that for every open set $U \subset X$, $f(U)$ is open in Y, but f is *not* continuous.
(4) Lee Exercise 2.20.
(5) Lee, Exercise 2.27.
(6) Lee, Exercise 2.28. (You may assume $\cos(x)$ and $\sin(x)$ are continuous functions, and you don't have to prove a is a bijection.)
(7) Lee, Exercise 2.35.
(8) Lee, Exercise 2.38.

Challenge problems (required for Math 531, optional for 431):

(9) Lee, Problem (not exercise) 2-5.
(10) Lee, Problem (not exercise) 2-10.

Bonus problems (not required for anyone):

(11) Define the *Zariski topology on* $\mathbb{R}^2$ as follows. A subset $C \subset \mathbb{R}^2$ is defined to be closed if there is a collection of polynomials $\{p_\alpha(x, y) \mid \alpha \in I\}$ so that
$$C = \{(x, y) \mid p_\alpha(x, y) = 0 \text{ for all } \alpha \in I\}.$$
  (a) Prove that this defines a topology on $\mathbb{R}^2$.
  (b) Prove that the Zariski topology is coarser than the usual, metric topology on $\mathbb{R}^2$.
  (c) Prove that the Zariski topology is not Hausdorff. (Hint: I think this is quite hard from the tools we've developed so far.)
  (d) Could we replace "there is a collection of polynomials" by "there is a single polynomial" in the definition of the Zariski topology? (Hint: This is also quite hard.)

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