MATH 431/531 FALL 2016
HOMEWORK 8
DUE NOVEMBER 16, 2016.

INSTRUCTOR: ROBERT LIPSHITZ

Problems:

(1) Use the fact that $\mathbb{R}$ is connected to prove that $\mathbb{R}$ has the least upper bound property.
(2) Lee, Exercise 4.38.
(3) Lee, Problem 4-1.
(4) Lee, Problem 4-2.
(5) Lee, Problem 4-3.
(6) Lee, Problem 4-4.
(7) Lee, Problem 4-12.

Challenge problems (required for Math 531, optional for 431):

(8) Given $a_0, \dots, a_n, b_0, \dots, b_n \in \mathbb{R}$ with $0 = a_0 < a_1 < \cdots < a_n = 1$, let $g_{a_0,\dots,a_n;b_0,\dots,b_n}$ be the piecewise-linear function defined on $[a_i, a_{i+1}]$ by

$$g(t) = b_i + \frac{b_{i+1} - b_i}{a_{i+1} - a_i}(t - a_i).$$

That is, for all i, $g(a_i) = b_i$ and g is linear on $[a_i, a_{i+1}]$. (I have changed notation slightly from Homework 5: I think this notation is less confusing.)

Prove: Given $f \in C^0[0,1]$ and $\epsilon > 0$ there is $n \in \mathbb{N}$ and $a_0, \dots, a_n, b_0, \dots, b_n \in \mathbb{Q}$ (with $0 = a_0 < a_1 < \cdots < a_n = 1$) so that $d_0(f, g_{a_0,\dots,a_n;b_0,\dots,b_n}) < \epsilon$. Here,

$$d_0(f,g) = \sup\{|f(x) - g(x)| \mid x \in [0,1]\}.$$

(9) Lee, Problem 4-21.

E-mail address: `lipshitz@uoregon.edu`