MATH 634 HOMEWORK 10
DUE NOVEMBER 28, 2018.

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Required problems, to turn in:

(1) Connecting homomorphisms…
  (a) Explain in words, to the extent possible, the geometry behind the connecting homomorphism in the homology long exact sequence for a triple, i.e., the map $H_n(X,Y) \to H_{n-1}(Y,Z)$ in the sequence
  $$\cdots \to H_n(Y,Z) \to H_n(X,Z) \to H_n(X,Y) \to H_{n-1}(Y,Z) \to \dots.$$
  associated to subspaces $Z \subset Y \subset X$.
  (b) Explain in words, to the extent possible, the geometry behind the connecting homomorphism in the cohomology long exact sequence for a pair, i.e., the map $H^{n-1}(A) \to H^n(X,A)$ in the sequence
  $$\cdots \to H^{n-1}(A) \to H^n(X,A) \to H^n(X) \to H^n(A) \to \cdots$$
  associated to a subspace $A \subset X$.

  (You don't have to prove your answers are correct.)

(2) Hatcher, 2.1.17.
(3) Hatcher, 2.1.18.
(4) Hatcher, 2.1.20. (The first half of this is an important observation.)
(5) Hatcher, 2.1.27.
(6) Hatcher, 2.1.29.
(7) Imitate our computation of $H_*(S^n)$ to compute the singular cohomology groups $H^*(S^n)$.

To think about but not turn in:

(1) Recall that we had previously defined $H^1(X) = [X, S^1]$ to be the set of homotopy classes of maps from X to S^1.
  (a) Given $A \subset X$, define $H^1(X,A)$ to be the set of homotopy classes of maps $X \to S^1$ sending A to $1 \in S^1$. Prove excision for $H^1(X,A)$. (You may add the hypothesis that $A \hookrightarrow X$ is a good inclusion if necessary.)
  (b) Define $H^0(X) = [X, \mathbb{Z}]$ to be the set of homotopy classes of maps $X \to \mathbb{Z}$, where $\mathbb{Z}$ has the discrete topology. The group structure on $\mathbb{Z}$ makes $H^0(X)$ into a group. Similarly, define $H^0(X,A) = [(X,A),(\mathbb{Z},\{0\})]$ to be the set of homotopy classes of maps sending A to 0. Prove there is an exact sequence
  $$0 \to H^0(X,A) \to H^0(X) \to H^0(A) \to H^1(X,A) \to H^1(X) \to H^1(A).$$

(2) Read the remaining problems in Hatcher section 2.1 and solve any that you don't know how to.

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