

MATH 634 HOMEWORK 3
DUE OCTOBER 10, 2018.

INSTRUCTOR: ROBERT LIPSHITZ

- (1) Hatcher 0.6 (pp. 18–20).
- (2) Hatcher Exercise 0.10 (p. 19).
- (3) Hatcher Exercise 0.11 (p. 19).
- (4) Hatcher Exercise 0.16 (p. 19).
- (5) Hatcher Exercise 0.20 (p. 19).
- (6) Recall that the *disjoint union topology* has the following property: given spaces X, Y, Z and continuous maps $f: X \rightarrow Z, g: Y \rightarrow Z$ there is a unique continuous map $h: X \amalg Y \rightarrow Z$ so that the following diagram commutes:

$$\begin{array}{ccc} X \amalg Y & \longleftarrow & Y \\ \uparrow & \searrow h & \downarrow g \\ X & \xrightarrow{f} & Z \end{array}$$

Here, the maps $X \rightarrow X \amalg Y$ and $Y \rightarrow X \amalg Y$ are the canonical inclusions. Moreover, this property characterizes the disjoint union topology. (If you are not familiar with this property, prove it.)

What operation has the analogous property for *based spaces* and *basepoint-preserving continuous maps*? (Hint: we saw it in class.) Formulate the property precisely and prove your answer satisfies it.

To think about but not turn in:

- (1) With Exercise 6 in mind, what are analogues $X \vee Y$ for other classes of mathematical objects (e.g., sets, groups, abelian groups, rings)? Are there (well-known) classes of objects for which \wedge does not exist?
- (2) Read through the remaining problems in Chapter 0, and do any that seem difficult or surprising.

E-mail address: lipshitz@uoregon.edu