MATH 634 HOMEWORK 7
DUE NOVEMBER 7, 2018.

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Required problems, to turn in:

(1) Hatcher 1.3.14 (p. 80)
(2) Hatcher 1.3.18 (p. 80)
(3) Hatcher 1.3.19 (p. 80)
(4) Given a group G, let $\mathrm{Hom}(G, \mathbb{Z})$ denote the set of group homomorphisms from G to $\mathbb{Z}$. Addition in $\mathbb{Z}$ (pointwise) makes $\mathrm{Hom}(G, \mathbb{Z})$ into an abelian group.
   Recall that we defined $H^1(X) = [X, S^1]$.
   Suppose that X is a path connected CW complex. Prove that $H^1(X) \cong \mathrm{Hom}(\pi_1(X, x_0), \mathbb{Z})$.
   (Hint: we may assume the 1-skeleton X^1 is a wedge of circles. There is a map $\Phi\colon H^1(X) \to \mathrm{Hom}(\pi_1(X, x_0), \mathbb{Z})$ defined by
   $$(f\colon X \to S^1) \mapsto (f_*\colon \pi_1(X, x_0) \to \pi_1(S^1)).$$
   Prove this is well-defined (basepoints!) and gives a group homomorphism. To prove injectivity, suppose $\Phi(f) = 0$ and lift f to the universal cover of S^1. To prove surjectivity, given a group homomorphism construct a corresponding map f starting with the 1-skeleton of X and then extending to the higher skeleta. Note that you can assume X has only one 0-cell, if you like, since both sides of the isomorphism are homotopy invariants.)

To think about but not turn in:

(1) In Problem 4, if X is not a CW complex then many things can go wrong. Give examples where the isomorphism fails if:
   (a) X is not semi-locally simply connected. (You can use a subspace of $\mathbb{R}^2$.)
   (b) X is not locally path connected. (Again, you can use a subspace of $\mathbb{R}^2$, but you probably aren't in a position to prove your answer.)
   (c) X is not Hausdorff. (You can use a space with finitely many points.)
(2) Let G be a graph, i.e., a 1-dimensional CW complex. Try to describe explicitly the universal cover of G, as a graph. Given such a description, extend the construction from class of the universal cover of a CW complex with a unique 0 cell to general connected CW complexes.
(3) Hatcher 1.3.23 (p. 81) is an important result; read it, and preferably prove it.
(4) Read the rest of the problems in Section 1.3 and solve any that seem interesting or challenging.

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