MATH 634 HOMEWORK 9
DUE NOVEMBER 21, 2018.

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Required problems, to turn in:

(1) Show that if X is path connected then $H^0(X) \cong \mathbb{Z}$.

(2) (Homotopy invariance of cohomology) Prove that if $f, g\colon X \to Y$ are homotopic maps then $f^* = g^*\colon H^n(Y) \to H^n(X)$. Conclude that if $X \simeq Y$ then $H^n(X) \cong H^n(Y)$ for each n.

(3) (Excision for cohomology) Recall that for $A \subset X$,

$$C^n(X, A) = \{c \in C^n(X) \mid c(\sigma) = 0 \text{ for all } \sigma\colon \Delta^n \to A\}.$$

  (a) Prove that $C^*(X, A)$ is a subcomplex of $C^*(X)$. So, we can define $H^n(X, A) = \ker(\delta|_{C^n(X,A)})/\mathrm{Im}(\delta|_{C^{n-1}(X,A)})$.

  (b) Suppose $Z \subset A \subset X$ with the closure of Z contained in the interior of A. Prove that $H^n(X, A) \cong H^n(X \setminus Z, A \setminus Z)$. (This isomorphism is induced by the inclusion map $(X \setminus Z, A \setminus Z) \hookrightarrow (X, A)$, though you don't have to prove that if you don't want to.)

(4) Hatcher 2.1.12 (p. 132)

(5) Let X be a simplicial complex and $A \subset X$ a subcomplex.

  (a) Define the relatively *simplicial* homology and cohomology groups $H_n^{\mathrm{simp}}(X, A)$ and $H^n_{\mathrm{simp}}(X, A)$.

  (b) Compute the simplicial homology and cohomology groups for the pair (X, A) where X is a 2-simplex (viewed as a simplicial complex with 3 0-simplices, 3 1-simplices, and 1 2-simplex) and A is:

    (i) A single vertex.

    (ii) Two vertices.

    (iii) An edge (and the two vertices at its ends).

    (iv) The entire boundary of the 2-simplex.

To think about but not turn in:

(1) Hatcher 2.1.26 (p. 133).

(2) Hatcher 2.1.29 (p. 133). (Use a Δ-complex to perform the computation, say.)

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