MATH 636 HOMEWORK 3
DUE APRIL 17, 2019.

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Problems to turn in:

(P-1) Hatcher 4.1.4 (p. 358).
(P-2) Hatcher 4.1.11 (pp. 358–359).
(P-3) Hatcher 4.1.12 (p. 359).
(P-4) Hatcher 4.1.16 (p. 359).
(P-5) Hatcher 4.1.17 (p. 359).
(P-6) (a) Suppose that M is a closed, connected m-manifold. Suppose further that M is *triangulated*, i.e., is an m-dimensional simplicial complex. Suppose $p \in M$ is on a codimension-1 face (or facet) $\partial_i \sigma$ of some simplex σ. Show that p is on a codimension-1 face of exactly two simplices σ, σ'.
(b) With notation as in the previous part, let $\alpha = \sum \sigma_i \in C_m(M; \mathbb{F}_2)$ be the sum of the m-simplices in M, viewed, via their characteristic maps, as maps $\sigma_i \colon \Delta^m \to M$. Show that α is a cycle.
(c) Show that for any point p in the interior of some m-simplex σ_i, the image of α in $H_m(M, M \backslash \{p\}; \mathbb{F}_2)$ is a generator. Deduce that α is the generator of $H_m(M; \mathbb{F}_2) \cong \mathbb{F}_2$ so α is a (in fact, the) mod-2 fundamental class for M.
(d) Now, suppose that $M^m \subset N^n$ is a closed submanifold of a manifold N, and that N is triangulated in such a way that $M \subset N$ is a subcomplex. Show that $i_*[M] \in H_m(N; \mathbb{F}_2)$, the homology class represented by M, is the sum of the m-simplices in N which are contained in M. (Hint: this is easy.)

Review / qualifying exam practice (not to turn in):

(Q-1) Hatcher 4.1.3, 4.1.5, 4.1.8, 4.1.13, 4.1.14,

More problems to think about but not turn in:

(OP-1) Hatcher 4.1.10, 4.1.18.
(OP-2) Extend Problem (P-6) to the case that M is an n-dimensional CW complex and $\alpha \in C_n^{\text{cell}}(M; \mathbb{F}_2)$ is the sum of the n-cells in M. (You'll have to follow α through the isomorphism between cellular and singular homology.)
(OP-3) Extend Problem (P-6) to $\mathbb{Z}$-coefficients.

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