MATH 636 HOMEWORK 4
DUE APRIL 24, 2019.

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Problems to turn in:

(P-1) Hatcher 4.1.19 (p. 359).
(P-2) Hatcher 4.1.20 (p. 359).
(P-3) Hatcher 4.2.5 (p. 389).
(P-4) Hatcher 4.2.7 (p. 389). (Hint: build the universal cover of X.)

Review / qualifying exam practice (not to turn in):

(Q-1) Adapt the proof from class to prove the injectivity statement in the excision theorem for homotopy groups.

More problems to think about but not turn in:

(OP-1) Suppose that M is a smooth, n-dimensional manifold, with tangent bundle TM. Define a vector bundle $\Lambda^n TM$ whose fiber over a point $x \in M$ is $\Lambda^n T_x M$, the top exterior power of the tangent space at x. ("Define" means you should write down a precise definition of such a bundle, not that the previous sentence is a precise definition.) There is an associated covering space $(\Lambda^n TM \setminus 0)/\mathbb{R}_+$ obtained by deleting the 0-section and quotienting by scaling by positive real numbers. Show that $(\Lambda^n TM \setminus 0)/\mathbb{R}_+$ is the orientation double cover of M.

(OP-2) Given a manifold M, give a natural definition of an element $w \in H^1(M; \mathbb{F}_2)$ so that $w = 0$ if and only if M is orientable. (If you want "natural" to be precise, w is natural with respect to codimension-0 embeddings of (open) submanifolds.)

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