Problems to turn in:
(P-1) Hatcher 4.2.22 (p. 390).
(P-2) The case \( n > 1 \) of Hatcher 4.2.23 (p. 390). (You can view the \( n = 1 \) case as an optional exercise.)
(P-3) Hatcher 4.2.31 (p. 392). (You may assume the nullhomotopy of the inclusion \( F \hookrightarrow E \) is constant on \( \tilde{x}_0 \).)
(P-4) Hatcher 4.B.2 (p. 428). Note: you solved part of this in a previous homework. You don’t have to re-prove that part; just cite it.

Review / qualifying exam practice (not to turn in):
(Q-1) Hatcher 4.2.28
(Q-2) Hatcher 4.2.29.
(Q-3) Hatcher 4.3.34.
(Q-4) Hatcher 4.B.1 (p. 428).
(Q-5) Give a direct proof of exactness for the long exact sequence
\[
\cdots \to \pi_n(F, \tilde{x}_0) \to \pi_n(E, \tilde{x}_0) \to \pi_n(B, x_0) \to \pi_{n-1}(F, \tilde{x}_0) \to \cdots
\]
associated to a fibration \( F \to E \to B \).

More problems to think about but not turn in:
(OP-1) In this problem, we show that Hopf’s original definition of the Hopf invariant is a homotopy invariant. The problem assumes a little more knowledge of transversality than we have covered in class.
(a) Let \( f: S^3 \to S^2 \) be a smooth map and \( p, q, r \) regular values of \( f \). Show that \( \operatorname{lk}(f^{-1}(p), f^{-1}(q)) = \operatorname{lk}(f^{-1}(p), f^{-1}(r)) \). (Hint: consider a path \( \gamma \) from \( q \) to \( r \) with the property that \( f \) is transverse to \( \gamma \).)
(b) Show that every continuous map \( S^3 \to S^2 \) is homotopic to a smooth map, and that if two smooth maps are homotopic then they are homotopic via a smooth map \( S^3 \times [0, 1] \to S^2 \).
(c) Now, suppose that \( f, g: S^3 \to S^2 \) are homotopic smooth maps and \( p, q \) are regular values of both \( f \) and \( g \). Show that \( \operatorname{lk}(f^{-1}(p), f^{-1}(q)) = \operatorname{lk}(g^{-1}(p), g^{-1}(q)) \).
(d) Given a continuous map \( f: S^3 \to S^2 \), let \( \tilde{f} \) be a smooth map homotopic to \( f \) and \( p, q \) regular values of \( \tilde{f} \). Prove that
\[
\operatorname{lk}(\tilde{f}^{-1}(p), \tilde{f}^{-1}(q))
\]
is an invariant of the homotopic class of \( f \).

(OP-2) Let \( K_1, K_2 \subset S^3 \) be knots (smoothly embedded circles) and \( \Sigma_1, \Sigma_2 \subset B^4 \) smoothly embedded, orientable surfaces with \( \partial \Sigma_i = K_i \). Show that \( \operatorname{lk}(K_1, K_2) = \pm \# \Sigma_1 \cap \Sigma_2 \).
(Optionally, note that orientations of the \( K_i \) induce orientations of the \( \Sigma_i \), and the sign in the formula becomes +.)
(OP-3) Let $M^{n-1}, N^{n-1} \subset S^{2n-1}$ be closed, orientable submanifolds. Suppose that $M = \partial P$ for some $n$-dimensional manifold-with-boundary $P \subset S^{2n-1}$. Show that if $N \pitchfork P$ then $\text{lk}(M,N) = \pm \#P \cap N$.

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