MATH 636 HOMEWORK 7
DUE MAY 15, 2019.

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Problems to turn in:

(P-1) Hatcher 4.2.34 (p. 392).

(P-2) Hatcher 4.2.37 (p. 392).

(P-3) Recall that $SO(3) \cong \mathbb{R}P^3$. Compute $\pi_i(SO(3))$ and $\pi_i(SO(4))$ for $i \leq 3$. Similarly, use the fact that $SU(2) \cong S^3$ to compute $\pi_i(SU(2))$ for $i \leq 5$, $\pi_i(SU(3))$ for $i \leq 3$, and prove $\pi_5(SU(3))$ is nontrivial. (You may take as given the stable homotopy groups of spheres listed in Hatcher, but be careful about what the stable range is.)

(P-4) Prove that any vector bundle over a compact, contractible space X is trivial. (The same is true for paracompact base spaces.)

(P-5) How many isomorphism classes of k-dimensional, real vector bundles over S^1 are there for each k? Over S^2? Over S^3?

Review / qualifying exam practice (not to turn in):

(Q-1) Hatcher 4.2.14, 4.2.19, 4.2.30.

More problems to think about but not turn in:

(OP-1) Recall that the orthogonal $O(k)$ acts on $V_k(\mathbb{R}^n)$ on the right, with quotient $\mathrm{Gr}_k(\mathbb{R}^n)$. Of course, $O(k)$ also acts on $\mathbb{R}^k$, on the left, by matrix multiplication. Let

$$E = V_k(\mathbb{R}^n) \times \mathbb{R}^k / \sim$$

where $(xA, v) \sim (x, Av)$ for all $A \in O(k)$. Prove that addition and scalar multiplication in $\mathbb{R}^k$ make E into a vector bundle, and in fact E is isomorphic to the canonical bundle over $\mathrm{Gr}_k(\mathbb{R}^n)$.

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