Update 4/1/20: Fixed typo in problems 9 and 10.

Reminder. There is also online homework. The first online homework is also due Monday, April 6.

Required problems: (hand these in):

1. Let $S = \{at^2 + b \mid a, b \in \mathbb{R}\}$. Is $S$ a subspace of $\mathbb{P}_2$? Justify your answer.
2. Let $S = \{at^2 + 2 \mid a \in \mathbb{R}\}$. Is $S$ a subspace of $\mathbb{P}_2$? Justify your answer.
3. Let $W$ be the set of vectors in $\mathbb{R}^3$ of the form $[2a + b, 3b - a, a + b]^T$. Find vectors $\vec{u}$ and $\vec{v}$ so that $W = \text{Span}\{\vec{u}, \vec{v}\}$. Why does this imply that $W$ is a subspace of $\mathbb{R}^3$?
4. Let $W$ be the set of vectors in $\mathbb{R}^3$ of the form $[2a + b, 3b - a - 1, a + b - 2]^T$. Is $W$ a subspace of $\mathbb{R}^3$? If so, find vectors $\vec{u}$ and $\vec{v}$ so that $W = \text{Span}\{\vec{u}, \vec{v}\}$. If not, give an example showing that $W$ is not a subspace.
5. Complete the following proof that if $c\vec{u} = \vec{0}$ and $c \neq 0$ then $\vec{u} = \vec{0}$:

\[
c\vec{u} = \vec{0} \quad \text{Given.}
\]
\[
(1/c)(c\vec{u}) = (1/c)\vec{0} \quad \text{Scalar multiplication is well-defined}
\]
\[
((1/c)(c))\vec{u} = (1/c)\vec{0} \quad \text{By axiom } \text{__________}(a)
\]
\[
(1)\vec{u} = (1/c)\vec{0}
\]
\[
(1)\vec{u} = \vec{0} \quad \text{By Problem 28}
\]
\[
\vec{u} = \vec{0} \quad \text{By axiom } \text{__________}(b)
\]

6. Let

\[
A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \end{bmatrix}.
\]

Find a (finite) list of vectors which span the null space of $A$.

7. Let

\[
W = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \mid a + c = b + d \right\},
\]

with the usual operations of vector addition and scalar multiplication. Use a theorem from class / Section 4.2 to show that $W$ is a vector space.
(8) Let

\[
A = \begin{bmatrix}
3 & -6 \\
2 & -4 \\
5 & -10
\end{bmatrix}
\]

Find a non-zero vector in the column space of \( A \) and a non-zero vector in the null space of \( A \).

(9) Consider the system of equations

\[
\begin{align*}
-1x_1 - 1x_2 + 4x_3 + 5x_4 + 0x_5 - 13x_6 &= 0 \\
-1x_1 - 2x_2 + 8x_3 + 6x_4 - 5x_5 - 7x_6 &= 0 \\
2x_1 + 8x_2 - 31x_3 - 16x_4 + 27x_5 - 5x_6 &= 0 \\
3x_1 + 11x_2 - 41x_3 - 22x_4 + 33x_5 + 0x_6 &= 0 \\
x_1 + 2x_2 - 7x_3 - 5x_4 + 5x_5 + 4x_6 &= 0 \\
x_1 - 5x_2 + 16x_3 + 7x_4 - 5x_5 - 10x_6 &= 0.
\end{align*}
\]

It can be shown that \( (x_1, x_2, x_3, x_4, x_5, x_6) = (1, -2, -1, -2, -2, -1) \) is a solution. Use material from this week (or Section 4.2 in the book) to explain why \( (x_1, x_2, x_3, x_4, x_5, x_6) = (5, -10, -5, -10, -10, -5) \) is another solution. (Do not do any nontrivial computations.)

(10) Consider the system of equations

\[
\begin{align*}
-1x_1 - 1x_2 + 4x_3 + 5x_4 + 0x_5 &= -13 \\
-1x_1 - 2x_2 + 8x_3 + 6x_4 - 5x_5 &= -7 \\
2x_1 + 8x_2 - 31x_3 - 16x_4 + 27x_5 &= -5 \\
3x_1 + 11x_2 - 41x_3 - 22x_4 + 33x_5 &= 0 \\
x_1 + 2x_2 - 7x_3 - 5x_4 + 5x_5 &= 4 \\
x_1 - 5x_2 + 16x_3 + 7x_4 - 5x_5 &= -10.
\end{align*}
\]

It can be shown that this system has a solution. Explain (using recent material) why the system

\[
\begin{align*}
-1x_1 - 1x_2 + 4x_3 + 5x_4 + 0x_5 &= 26 \\
-1x_1 - 2x_2 + 8x_3 + 6x_4 - 5x_5 &= 14 \\
2x_1 + 8x_2 - 31x_3 - 16x_4 + 27x_5 &= 10 \\
3x_1 + 11x_2 - 41x_3 - 22x_4 + 33x_5 &= 0 \\
x_1 + 2x_2 - 7x_3 - 5x_4 + 5x_5 &= -8 \\
x_1 - 5x_2 + 16x_3 + 7x_4 - 5x_5 &= 20.
\end{align*}
\]

must also have a solution.

(11) Define \( T: \mathbb{P}_2 \to \mathbb{R}^2 \) by \( T(p(t)) = \begin{bmatrix} p(1) \\ p(2) \end{bmatrix} \).

(a) Show that \( T \) is a linear transformation.

(b) Find a polynomial \( p(t) \) that spans the kernel of \( T \).

(c) Show that the image (book: range) of \( T \) is all of \( \mathbb{R}^2 \).
(12) Consider the set of vectors
\[
\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \subset \mathbb{R}^3.
\]

(a) Does this set span \(\mathbb{R}^3\)? Justify.
(b) Is this set linearly independent? Justify.
(c) Is this set a basis for \(\mathbb{R}^3\)? Justify.

(13) Consider the polynomials \(p_1(t) = 1 + 2t\), \(p_2(t) = -1 + 2t\), and \(p_3(t) = t\).
By inspection, write a linear dependence among \(p_1(t)\), \(p_2(t)\), and \(p_3(t)\).
Then find a basis for \(\text{Span}\{p_1(t), p_2(t), p_3(t)\}\).

(14) Let \(\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\} \).

(a) Suppose \([\vec{x}]_\mathcal{B} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}\). Find \(\vec{x}\).
(b) Let \(\vec{y} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}\). Find \([\vec{y}]_\mathcal{B}\).

(15) Find a basis for the subspace from Problem 3 and find its dimension.
(16) Find the dimensions of the subspaces from Problems 1 and 13.
(17) Let
\[
A = \begin{bmatrix} 1 & 2 & -7 \\ 0 & 1 & -4 \\ 2 & 2 & -6 \end{bmatrix}
\]
Find bases for \(\text{Null}(A)\) and \(\text{Col}(A)\)

Suggested practice (don’t hand these in):
- Please read and make sure you can do the practice problems in sections 4.1–4.5.
- Please read and use for review problems 4.1.23(a–d), 4.1.24(a–c), 4.2.25, 4.2.26, 4.3.21, 4.3.22, 4.4.15, 4.4.16, 4.5.19, 4.5.20.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem.
Similar problems:

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Blog. Optional: Follow the steps in the post “Getting Started” on the 342 in CoCalc, Spring 2020 blog. The blog is at:

https://blogs.uoregon.edu/math342sp20lipshitz/

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