MATH 342
WRITTEN HOMEWORK 3
DUE APRIL 20, 2020.

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Required problems: (hand these in):

(1) Let
$$A = \begin{bmatrix} 14 & 30 \\ -5 & -11 \end{bmatrix}.$$
Is 3 an eigenvalue of A? What about 4?

(2) Let
$$A = \begin{bmatrix} 8 & 10 \\ -5 & -7 \end{bmatrix}.$$
Is $[1, 1]^T$ an eigenvector of A? If so, find the corresponding eigenvalue. Is $[1, -1]^T$ an eigenvector of A? If so, find the corresponding eigenvalue.

(3) Let
$$A = \begin{bmatrix} -3 & 8 \\ -4 & 9 \end{bmatrix}.$$
The eigenvalues of A are 1 and 5. Find a basis for the corresponding eigenspaces.

(4) Let
$$A = \begin{bmatrix} 0 & 13 & 32 & -24 \\ -2 & -3 & -4 & 12 \\ 0 & 4 & 10 & -8 \\ -1 & 1 & 5 & 1 \end{bmatrix}.$$
The eigenvalues of A are 1, 2, and 3. Find bases for the corresponding eigenspaces.

(5) Let
$$A = \begin{bmatrix} 3 & 1 & 4 \\ 3 & 1 & 4 \\ 3 & 1 & 4 \end{bmatrix}.$$
Find one eigenvalue of A without doing any calculation. Justify your answer.

(6) Explain why a 3×3 matrix can have at most three distinct eigenvalues.

(7) Suppose A is an invertible matrix and λ is an eigenvalue of A. Show that $1/\lambda$ is an eigenvalue of A^{-1}. (Hint: your argument will start "If λ is an eigenvalue of A then there is a nonzero vector $\vec{v}$ so that. . . .")

(8) Let
$$A = \begin{bmatrix} -10 & 12 \\ -6 & 8 \end{bmatrix}$$
Find the characteristic polynomial and eigenvalues of A.

(9) Find the characteristic polynomial of

$$\begin{bmatrix} -1 & 8 & 2 \\ -1 & 5 & 1 \\ 1 & -4 & 0 \end{bmatrix}.$$

(10) Suppose an $n \times n$ matrix A has n real eigenvalues $\lambda_1, \dots, \lambda_n$. So,

$$det(A - \lambda I) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda).$$

Explain why $\det(A) = \lambda_1 \cdots \lambda_n$, the product of the eigenvalues of A.

Remark. If we consider also complex eigenvalues, as we will, a version of this statement is true for any square matrix.

(11) Show that A and A^T have the same eigenvalues. (Hint: how are their characteristic polynomials related?)

(12) Let $A = PDP^{-1}$ where

$$P = \begin{bmatrix} -1 & -5 \\ 1 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

Compute A and A^5.

(13) Let

$$A = \begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix}.$$

Diagonalize A. That is, find matrices P and D so that $A = PDP^{-1}$ (or equivalently, $D = P^{-1}AP$).

(14) One of the roots of the characteristic polynomial you found in Problem 9 is 2. Diagonalize

$$\begin{bmatrix} -1 & 8 & 2 \\ -1 & 5 & 1 \\ 1 & -4 & 0 \end{bmatrix}.$$

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in sections 5.1–5.3.
- Please read and use for review problems 5.1.21, 5.1.22, 5.2.21, 5.2.22, and 5.3.21.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem.

Similar problems:

HW Problems	Similar textbook problems
1–2	5.1.1–8
3–4	5.1.9–16
5	5.1.19–20
6	5.1.23–24
7	5.1.25–30
8	5.2.1–8
9	5.2.9–14
10	5.2.19
11	5.2.20
12	5.3.1–4
13–14	5.3.7–20

Blog. Optional:

- Follow the steps in the post "Eigenvectors and Eigenvalues" on the *342 in CoCalc, Spring 2020* blog. The blog is at: `https://blogs.uoregon.edu/math342sp20lipshitz/`
- Use Sage to check your work on problems 1, 9, and 14.
- How big a matrix can CoCalc (Sage) find eigenvectors / eigenvalues for in reasonable time? Do some experiments. You might find the `random_matrix` command useful for this. (You can also use the `time` command to time your computations, if you like.)

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