MATH 342
WRITTEN HOMEWORK 4
DUE APRIL 27, 2020.

INSTRUCTOR: ROBERT LIPSHITZ

Required problems (hand these in):

(1) Let V be a vector space and $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ an ordered basis for V. Suppose that $T\colon V \to \mathbb{R}^2$ is a linear transformation so that

$$T(x_1\vec{b}_1 + x_2\vec{b}_2 + x_3\vec{b}_3) = \begin{bmatrix} x_1 - x_2 + 2x_3 \\ 3x_1 - 5x_2 \end{bmatrix}.$$

Find the matrix for T with respect to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.

(2) Let $T\colon \mathbb{P}_2 \to \mathbb{P}_4$ be the transformation

$$T(p(t)) = (t+3)p(t) + p'(t).$$

(a) Compute $T(2 + 3t + 4t^2)$.
(b) Show that T is a linear transformation.
(c) Find the matrix for T with respect to the ordered bases $\{1, t, t^2\}$ for $\mathbb{P}_2$ and $\{1, t, t^2, t^3\}$ for $\mathbb{P}_3$.

(3) Define $T\colon \mathbb{P}_1 \to \mathbb{R}_2$ by

$$T(p(t)) = \begin{bmatrix} p(0) \\ p(1) \\ p(2) \end{bmatrix}.$$

(a) Compute $T(2 + 3t)$.
(b) Show that T is a linear transformation.
(c) Find the matrix for T with respect to the ordered basis $\{1, t\}$ for $\mathbb{P}_1$ and the standard basis for $\mathbb{R}^3$.

(4) Define $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ by

$$T(\vec{x}) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \vec{x}.$$

Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}.$$

Find the matrix $[T]_{\mathcal{B}}$ for T with respect to $\mathcal{B}$.

(5) Define $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ by

$$T(\vec{x}) = \begin{bmatrix} -7 & 18 \\ -3 & 8 \end{bmatrix} \vec{x}.$$

Find a basis $\mathcal{B}$ so that $[T]_{\mathcal{B}}$ is diagonal (and find $[T]_{\mathcal{B}}$).

(6) Let $V = \{a_0 + a_1 e^x + a_2 e^{2x} + a_3 e^{3x} \mid a_1, a_2, a_3 \in \mathbb{R}\}$, which is a subspace of the vector space of functions $\mathbb{R} \to \mathbb{R}$. Let $T\colon V \to V$ be the linear transformation $T(f(x)) = f''(x)$. What are the eigenvalues of T? Find an eigenvector corresponding to each eigenvalue.

(7) Consider the linear transformation $T\colon \mathbb{P}_2 \to \mathbb{P}_2$ given by $T(p(x)) = p(0) + p(1) + p'(x) + 3x^2 p''(x)$. Let $\mathcal{B}$ be the basis $\{1, x, x^2\}$ for $\mathbb{P}_2$.
   (a) Find the matrix A for T with respect to the basis $\mathcal{B}$.
   (b) Find the eigenvalues of A, and a basis for $\mathbb{R}^3$ consisting of eigenvectors of A.
   (c) Find a basis for $\mathbb{P}_2$ consisting of eigenvectors for T.

(8) Let
$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}.$$
Find the eigenvalues of A (which may be complex numbers) and a basis for each eigenspace in $\mathbb{C}^2$.

(9) Let
$$A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}.$$
Find the eigenvalues of A (which may be complex numbers) and a basis for each eigenspace in $\mathbb{C}^2$.

(10) The matrix
$$\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$
corresponds to the composition of a rotation and a scaling. Give the angle ϕ of the rotation and the scale factor r.

(11) Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$. Compute A^{20} (by hand).

(12) Plot the following complex numbers in the complex plane, on one graph: $1+i$, $2-3i$, $2e^{\frac{\pi}{4}i}$, $2e^{\frac{\pi}{2}i}$, $2e^{\frac{3\pi}{4}i}$, $2\cos(\pi/3) + 2i\sin(\pi/3)$.

(13) Plot the following complex numbers in the plane, on one graph: $(1 + i\sqrt{3})$, $(1+i\sqrt{3})^2$, $(1+i\sqrt{3})^3$, $(1+i\sqrt{3})^4$, $e^{\frac{\pi}{3}i}$, $e^{\frac{2\pi}{3}i}$, $e^{\frac{3\pi}{3}i}$, $e^{\frac{4\pi}{3}i}$, $e^{\frac{5\pi}{3}i}$.

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 5.4 and 5.5.
- If you have taken a course on writing proofs, try problems 5.4.25, 5.4.26, 5.4.19, 5.4.20, 5.4.21, 5.4.23, 5.5.23, and 5.5.24.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Similar problems:

HW Problems	Similar textbook problems
1–3	5.4.1–10
4	5.4.11–12
5	5.4.13–16
8–9	5.5.1–6
10	5.5.7–12

Blog. Optional:

- Work through the blog post "Complex numbers".
- Use CoCalc to check your answers to problems 4, 8, 9, 12, and 13.

Email address: lipshitz@uoregon.edu