MATH 342
WRITTEN HOMEWORK 6
DUE MAY 11, 2020.

INSTRUCTOR: ROBERT LIPSHITZ

Updated: fixed a typo in Problem 4c.
Required problems (hand these in):

(1) Determine whether each of the following sets of vectors is orthonormal, orthogonal but not orthonormal, or not orthogonal.

(a)
$$\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}$$

(b)
$$\begin{bmatrix}2/3\\2/3\\1/3\end{bmatrix}, \begin{bmatrix}-2/3\\1/3\\2/3\end{bmatrix}, \begin{bmatrix}1/3\\-2/3\\2/3\end{bmatrix}$$

(c)
$$\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$$

(2) Compute the orthogonal projection of $\vec{v} = \begin{bmatrix}2\\3\end{bmatrix}$ onto the line L through $\begin{bmatrix}-1\\4\end{bmatrix}$ and the origin. Then write $\vec{v}$ as the sum of a vector in L and a vector orthogonal to L.

(3) Consider the pair of vectors
$$\vec{u}_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, \ \vec{u}_2 = \begin{bmatrix}1\\0\\-1\end{bmatrix}.$$

(a) Normalize these vectors to obtain an orthonormal set.

(b) Find the orthogonal projection of $\vec{v} = \begin{bmatrix}3\\1\\4\end{bmatrix}$ onto the subspace $\mathrm{Span}(\{\vec{u}_1, \vec{u}_2\})$.

(4) Let
$$\vec{y} = \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \qquad \vec{v}_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}.$$

(a) Find the closest point to $\vec{y}$ in $\mathrm{Span}(\{\vec{v}_1, \vec{v}_2\})$.
(b) Write $\vec{y}$ as the sum of a vector in $\mathrm{Span}(\{\vec{v}_1, \vec{v}_2\})$ and a vector orthogonal to $\mathrm{Span}(\{\vec{v}_1, \vec{v}_2\})$.
(c) Find the best approximation to $\vec{y}$ by vectors of the form $c_1\vec{v}_1+c_2\vec{v}_2$. What are c_1 and c_2?

(5) Consider the subspace

$$W = \mathrm{Span}\left(\left\{\begin{bmatrix}4\\4\\7\end{bmatrix}, \begin{bmatrix}5\\3\\7\end{bmatrix}\right\}\right) \subset \mathbb{R}^3.$$

Apply the Gram-Schmidt process to find an orthonormal basis for W.

(6) Find an orthonormal basis for the column space of the following matrix:

$$\begin{bmatrix}4 & 3 & 5\\ 2 & 4 & 5\\ 2 & 0 & -4\\ 1 & 5 & 3\end{bmatrix}$$

(7) Let $\{\vec{v}_1, \dots, \vec{v}_k\}$ be an orthonormal basis for a subspace W of $\mathbb{R}^n$. Define $T\colon \mathbb{R}^n \to \mathbb{R}^n$ to be $T(\vec{x}) = \mathrm{proj}_W(\vec{x})$. Show that T is a linear transformation. (Hint: start by writing down the formula for T.)

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 6.2, 6.3, and 6.4.
- Use Exercises 6.2.23, 6.2.24, 6.3.21, 6.3.22, 6.4.17(a,b), 6.4.18(a,b) for review.
- If you have some experience writing proofs, try Exercises 6.2.28, 6.2.29, 6.2.33.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Similar problems:

HW Problems	Similar textbook problems
1	6.2.1–6
2	6.2.11–16
3	6.2.17–22, 6.3.3–6
4	6.3.7–14
5	6.4.1–8
6	6.4.9–12
7	6.4.22

Blog. Optional:

- Work through the blog post "Dot Products and Orthogonality".
- Use CoCalc to check your answers to the problems above (except for Problem 7).
- Write a function in CoCalc (Sage) which computes the orthogonal projection of $\vec{v}$ to Span($\vec{w}$), where $\vec{v}$ and $\vec{w}$ are vectors in the same $\mathbb{R}^n$. (Your function should take two inputs, $\vec{v}$ and $\vec{w}$.) Test your function in some examples. Only slightly harder: write a function which computes the orthogonal projection of $\vec{v}$ onto Span($\vec{w}_1, \dots, \vec{w}_k$). (In the second case, start by applying Sage's Gram-Schmidt to $\vec{w}_1, \dots, \vec{w}_k$.)

Email address: `lipshitz@uoregon.edu`