MATH 342
WRITTEN HOMEWORK 8
DUE MAY 26, 2020.

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Required problems (hand these in):

(1) Which of the following matrices are symmetric?
   (a) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 7 & 9 \end{bmatrix}$

(2) Which of the following matrices are orthogonal? If orthogonal, find the inverse:
   (a) $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$
   (b) $\begin{bmatrix} 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \end{bmatrix}$

(3) For each of the following, find an orthogonal matrix P and a diagonal matrix D so that $A = PDP^T = PDP^{-1}$. Feel free to use a calculator or software to help with computing characteristic polynomials and with row-reduction.
   (a) $A = \begin{bmatrix} -6 & 12 \\ 12 & 1 \end{bmatrix}$
   (b) $A = \begin{bmatrix} -5 & 24 & 6 \\ 24 & -27 & 30 \\ 6 & 30 & 32 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 121 & 24 & -36 \\ 24 & 57 & -12 \\ -36 & -12 & 67 \end{bmatrix}$. (The eigenvalues in this case are 147, 49, and 49; you can take my word for this. If you're clever, you can solve this one with a little less work, if you start with the 147 eigenspace.)

(4) Compute the quadratic form $\vec{x}^T A\vec{x}$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.

(5) Find the 2×2 matrix for the quadratic form $2x_1^2 - 6x_1x_2 + 8x_2^2$.

(6) Find an orthogonal change of variables $\vec{x} = P\vec{y}$ which transforms the following quadratic forms into quadratic forms with no cross-terms (i.e., diagonalizes them), and write the new quadratic forms. Indicate if they are positive definite, positive semi-definite, negative definite, negative

semi-definite, or indefinite. Optional: check your work by plugging in the change of variables directly.

(a) $-6x_1^2 + 24x_1x_2 + x_2^2$.
(b) $-5x_1^2 + 48x_1x_2 + 12x_1x_3 - 27x_2^2 + 60x_2x_3 + 32x_3^2$.
(c) $121x_1^2 + 57x_2^2 + 67x_3^3 + 48x_1x_2 - 72x_1x_3 - 24x_2x_3$.

(7) Let

$$Q(x_1, x_2, x_3) = -5x_1^2 + 48x_1x_2 + 12x_1x_3 - 27x_2^2 + 60x_2x_3 + 32x_3^2.$$

Find:

(a) the maximum value of $Q(x_1, x_2, x_3)$ subject to the constraint $x_1^2 + x_2^2 + x_3^2 = 1$.
(b) The unit vector $\vec{u} = [x_1, x_2, x_3]^T$ where this maximum is obtained.
(c) The maximum value of $Q(\vec{x})$ subject to the constraints $\vec{x} \cdot \vec{x} = 1$ and $\vec{x} \cdot \vec{u} = 0$.

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 7.1, 7.2, and 7.3.
- Use Exercise 7.1.25, 7.1.26, 7.2.21, 7.2.22, for review.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Similar problems:

HW Problems	Similar textbook problems
1	7.1.1–6
2	7.1.7–12
3	7.1.13–22
4	7.2.1, 2
5	7.2.3–6
6	7.2.9–18, 7.3.1, 2
7	7.3.3–6

Blog. No new blog entry this week. Sage has a huge amount of code related to quadratic forms (see `http://doc.sagemath.org/html/en/reference/quadratic_forms/`) but it's focused on number theory, and not well suited to what we've been studying. (Over $\mathbb{Z}$, quadratic forms are very complicated; over $\mathbb{R}$, they are simple. We are working over $\mathbb{R}$ in this course.) You already know how to use Sage to do the computations for this week.

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