MATH 342
WRITTEN HOMEWORK 9
DUE JUNE 1, 2020.

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Required problems (hand these in):

(1) Let
$$A = \begin{bmatrix} -5 & 11 \\ -10 & 2 \end{bmatrix}.$$
Find a singular value decomposition of A.

(2) Let
$$A = \begin{bmatrix} 3 & 6 & 6 \\ -4 & -8 & -8 \end{bmatrix}.$$
Find a singular value decomposition of A.

(3) For the matrix A in Problem 1 find a unit vector $\vec{u}$ so that $\|A\vec{u}\|$ is as big as possible.

(4) Prove: if A is a square matrix then $|\det(A)|$ is equal to the product of the singular values of A.

(5) Convert the following matrix to mean-deviation form, and construct the sample covariance matrix. Then find the principal components of the data. (Your answer to the last part will not be especially pretty; feel free to use a computer to find eigenvalues / eigenvectors, but if that's what you did, say so.)
$$\begin{bmatrix} 19 & 23 & 15 & 3 & 15 \\ 10 & 14 & 18 & 2 & 6 \end{bmatrix}$$

Suggested practice (don't hand these in):

- Please read and make sure you can do the practice problems in Sections 7.4 and 7.5.
- If you have some experience writing proofs, try exercises 7.4.18, 7.4.19, 7.4.21, 7.4.25.
- If you had trouble or got help with any of the assigned problems, solve another, similar problem (or two).

Similar problems:

HW Problems	Similar textbook problems
1, 2	7.4.5–12
3	7.4.14
4	7.4.17–20
5	7.5.1–4

Blog. Optional:
- Work through the blog post "Singular weather decomposition."
- Use the Sage to check your answers to the assigned problems.
- Create a 3×3 matrix A with rank 2. Add a small, random error to each entry; call the resulting matrix B. (The Sage command `random()` might help here.) What is the rank B? (Have Sage compute it for you.) How do the singular values of A and B compare? (Singular value decomposition, unlike rank, is numerically stable; this is also mentioned in the "numerical note" in Section 7.4 of the textbook. So, if you are working with real data, you always want to use the SVD (or something similar) rather than the rank.)
- Get some weather data of your own from the NOAA (as discussed in the blog post), or if you prefer some other data, take the singular value decomposition, and interpret the results. Add some written comments about the data you looked at and what you observed.

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