MATH 282 SPRING 2022
HOMEWORK 9
DUE JUNE 1, 2022.

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Required problems (quiz problems drawn from these):

- Section 16.8: 6, 9, 15.
- Section 16.9: 6, 13, 17, 27.
- Here is a kind of problem that seems to be missing from the book and the webworks database:
  (1) Let $C = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ be the unit circle and
  $$\vec{F} = \left\langle (x^2+y^2-1)e^{x^2y^3},\ \frac{x^4+2x^2y^2+y^4-1}{x^2+y^2+1} \right\rangle.$$
  Compute $\int_C \vec{F} \cdot \vec{n} ds$ (*not* $\vec{F} \cdot d\vec{T} ds = \vec{F} \cdot d\vec{r}$). (You might also like to use software to plot the vector field $\vec{F}$.)
  (2) Let $D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$. Compute
  $$\iint_D 2xe^{x^2y^3} + 2xy^3(x^2+y^2-1)e^{x^2y^3} + \frac{(4y^3+4x^2y)(x^2+y^2+1) - (x^4+2x^2y^2+y^4-1)(2y)}{(x^2+y^2+1)^2} dA$$
  (Hint: use the previous part.)
- Here's another example of the technique, in three dimensions:
  (1) Let $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1\}$ be the unit sphere and
  $$\vec{F} = \langle y\sin(xyz), -x\sin(xyz) - ze^{y^2+z^3}, ye^{y^2+z^3} \rangle$$
  Compute $\iint_S \vec{F} \cdot \vec{n} dS$. (Again, you might also like to use software to plot $\vec{F}$.)
  (2) Let $E = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \leq 1\}$ be the unit ball. Compute
  $$\iiint_E y^2z\cos(xyz) - x^2z\cos(xyz) - 2yze^{y^2+z^3} + 3yz^2e^{y^2+z^3} dV.$$
  (Hint: use the previous part.)

Suggested challenge problems (conceptually interesting problems in the text to think about, but which will not be on a quiz):

- Section 16.8: 19, 20.
- Section 16.9: 31, 32.

Practice matrix:

If you got help with these...	try these on your own for more practice.
16.8.6	16.8.1–5
16.8.9	16.8.7–12
16.8.15	16.8.13, 14
16.9.6, 13	16.9.5–14
16.9.17	16.9.18
16.9.27	16.9.25–30

Comments on some of these problems:

- 16.8.19: This is a common way that Stokes' Theorem is used in pure mathematics. Similarly, the identities in 16.8.20 get used fairly often when studying partial differential equations.
- 16.9.17: This is a classic (and useful) trick. It also shows up in the webworks problems.
- 16.9.31, 32: a classic. Expanding the hint in 16.9.31, you'll prove that the dot product of the left side with any vector $\vec{c}$ is equal to the dot product of the right side with $\vec{c}$.

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