The plan

We will choose three of the following groups of papers (democratically). We'll then divide up papers, so there's one presentation per day. Most papers will get one day only, but a few will get two. This has two goals: to practice reading and extracting ideas from papers in a reasonable amount of time, and to acquire a general knowledge of some important strains of thought in topology.

Some papers are necessary background for that group, and are marked with an asterisk. Others we can pick and choose between, based on interest. (I discuss more complicated dependencies when we divide up papers.) Survey papers which are intended as references but not for presentations are marked with plus signs.

We will probably divide up papers in a couple of rounds: in the first round, we'll divide up enough that everyone has one, and then a few weeks later we'll choose some more papers based on interest (which might change or become clearer after we've gotten started).

Note: each of these lists of papers is meant to be one reasonable path through some interesting material. I haven't made an effort to ensure that there's a paper by every key contributor to the development of the topic. There is some overlap between the different topics.

String Topology

String topology explores a surprising product structure discovered on the homology of the free loop space of a manifold, discovered in the 1990s, and various related operations.

- *M. Chas and D. Sullivan, "String topology." arXiv:math/9911159 (preprint).
  Gives a (somewhat imprecise) definition of the string product on the free loop space of a manifold.
- *R. Cohen and J. Jones, "A homotopy theoretic realization of string topology". *Math. Ann.* 324 (2002), no. 4, 773-798.
  Gives a precise definition of the string product, via an infinite-dimensional Pontrjagin-Thom type construction.
- R. Cohen and K. Gruher, String topology of Poincaré duality groups. *Groups, homotopy and configuration spaces*, 1-10. Geom. Topol. Monographs, 13.
  An analogue of the string product for the homology of LBG for G a Lie group.
- R. Cohen and V. Godin, "A polarized view of string topology." *Topology, geometry and quantum field theory*, 127-154. London Math Soc. Lecture Note Series, 308.
  Shows that the string topology of an n-dimensional manifold M forms a positive-boundary 2-dimensional topological field theory.

- R. Cohen, J. Klein, and D. Sullivan, “The homotopy invariance of the string topology loop product and string bracket.” *J. Topol.* 1 (2008), no 2, 391-408.
  Does what the title says.
- A. Abbondandolo and M. Schwarz, “Floer homology of cotangent bundles and the loop product.” *Geom. Topol.* 14 (2010), no. 3, 1569–1722.
  Shows the loop product agrees with the pair-of-pants product on the Floer homology of the cotangent bundle.
- N. Hingston and N. Wahl, “Product and coproduct in string topology.” arXiv:1709.06839 (preprint).
  Gives another construction of the string product, as well as a new coproduct.
- K. Cieliebak, N. Hingston, and A. Oancea, “Loop coproduct in Morse and Floer homology.” arXiv:2008.13168 (preprint).
  Relates the Hingston-Wahl coproduct and a pair-of-pants coproduct on the Floer homology of the cotangent bundle.
- V. Godin, “Higher string topology operations.” arXiv:0711.4859 (preprint).
  Shows that string topology inherits operations coming from the homology of the moduli space of Riemann surfaces.
- F. Naef, “The string coproduct ‘knows’ Reidemeister/Whitehead torsion.” arXiv:2106.11307 (preprint).
  Shows that, unlike the string product, the string coproduct distinguishes some homotopy equivalent manifolds.

Homotopy Coherence

Homotopy coherence is a framework for understanding structures that hold up to homotopy, like associativity for the product on the based loop space. This includes a variety of specific algebraic structures, like A_infty-algebras, and various general formulations including the theory of (infty,1) categories.

- J. Stasheff, “Homotopy associativity of H-spaces I, II”. *Trans. Amer. Math. Soc.* 108 (1963), 275-292 and 293–312.
  Defines A_n-spaces and algebras, respectively, spaces (algebras) which are associative up to a coherent family of homotopies; and proves that n-fold loop spaces are A_n-spaces, and their homologies are A_n-algebras (among other results).
- R. Vogt, “Homotopy limits and colimits.” *Math. Z.* 134 (1973), 11–52.
  Defines a notion of homotopy coherent diagrams of spaces and their (co)limits.
- J.-M. Cordier, “Sur la notion de diagramme homotopiquement cohérent.” Third Colloquium on Categories, Part VI (Amiens, 1980). *Cahiers Topologie Géom. Différentielle* 23 (1982), no. 1, 93–112.
  A reinterpretation of homotopy coherent diagrams, relevant to the modern perspective.

- J.-M. Cordier and T. Porter, “Vogt's theorem on categories of homotopy coherent diagrams.” *Math. Proc. Cambridge Philos. Soc.* 100 (1986), no. 1, 65–90.

  Cordier’s reinterpretation of homotopy coherent diagrams, and the relationship to strict diagrams. An alternative to the previous one.
- A. Joyal, “Quasi-categories and Kan complexes.” Special volume celebrating the 70th birthday of Professor Max Kelly. *J. Pure Appl. Algebra* 175 (2002), no. 1-3, 207–222.

  One of the first modern paper to take quasi-categories seriously, building on Boardman-Vogt, Cordier, etc.
- M. Kontsevich, “Feynman diagrams and low-dimensional topology.” *First European Congress of Mathematics*, Vol. II (Paris, 1992), 97–121.

  First half gives a relationship between A_\infty algebras (with inner products) and cohomology classes on the moduli space of curves. The second half discusses graph homology and Vassiliev invariants.
- M. Kontsevich, “Homological algebra of mirror symmetry.” *Proceedings of the International Congress of Mathematicians*, Vol. 1, 2 (Zürich, 1994), 120–139, Birkhäuser, Basel, 1995.

  Proposes homological mirror symmetry, a relationship between the A_∞-category of Lagrangian submanifolds of a symplectic manifold (the Fukaya category) and the derived category of coherent sheaves on its mirror.
- A. Polishchuk, “ Moduli of curves as moduli of A_∞-structures.” *Duke Math. J.* 166 (2017), no. 15, 2871–2924.

  Shows that a certain moduli space of curves corresponds to a moduli space of A_∞-structures on a particular algebra. This paper requires a substantial background in algebraic geometry.
- D. Dugger and D. Spivak, “Rigidification of quasi-categories.” *Algebr. Geom. Topol.* 11 (2011), no. 1, 225–261.

  A way of relating quasi-categories to the more familiar notion of simplicial categories. Requires at least a solid background in simplicial sets.

Contact Homology

Contact homology is a family of invariants of contact manifolds and their submanifolds. It has a wealth of applications to contact and smooth topology and connections to other areas of mathematics, like sheaf theory.

- *J. Etnyre, “Legendrian and transversal knots.” Handbook of knot theory, 105–185, Elsevier B. V., Amsterdam, 2005.

  A survey about Legendrian and transverse knots, two classes of knots related to contact topology.
- *Chekanov, Y. (2002). Differential algebra of Legendrian links. *Invent. Math.*, *150*(3), 441–483.

  Gives the first non-classical invariant of Legendrian knots and shows that it does, in fact, distinguish some with equal classical invariants. So, Legendrian knot theory is rich.

- *Y. Eliashberg, A. Givental, and H. Hofer, “Introduction to symplectic field theory.” GAFA 2000 (Tel Aviv, 1999). *Geom. Funct. Anal.* 2000, Special Volume, Part II, 560–673.
  First half gives an explanation of what contact homology is (or should be). The second half is tied to Gromov-Witten theory (and we won’t try to read it).
- *L. Ng, “Computable Legendrian invariants.” *Topology* 42 (2003), no. 1, 55–82.
  Gives a more computable formulation of Legendrian contact homology, for Legendrian knots in R^3.
- L. Ng, “ Knot and braid invariants from contact homology. I, II” *Geom. Topol.* 9 (2005), 247–297 and Geom. Topol. 9 (2005), 1603–1637. With an appendix by Siddhartha Gadgil.
  Uses Legendrian contact homology to define an invariant of smooth knots in 3-space.
- L. Ng, “Combinatorial knot contact homology and transverse knots.” *Adv. Math.* 227 (2011), no. 6, 2189–2219.
  Uses Legendrian contact homology to define an invariant of transverse knots in 3-space.
- L. Ng, D. Rutherford, V. Shende, S. Sivek, and E. Zaslow. “Augmentations are sheaves.” *Geom. Topol.* 24 (2020), no. 5, 2149–2286.
  Extracts an A-infinity category from Legendrian contact homology and shows that it is related to a category of constructible sheaves, proving a conjecture of Shende-Treumann-Zaslow.
- T. Ekholm, J. Etnyre, and M. Sullivan, “The contact homology of Legendrian submanifolds in R^{2n+1}”. *J. Differential Geom.* 71 (2005), no. 2, 177–305.
  An analytic construction of Legendrian contact homology.
- M. Hutchings and J. Nelson, “Cylindrical contact homology for dynamically convex contact forms in three dimensions.” *J. Symplectic Geom.* 14 (2016), no. 4, 983–1012.
  Constructs contact homology for contact structures on closed 3-manifolds, under some hypotheses. This paper is somewhat technical.
- J. Pardon, “Contact homology and virtual fundamental cycles.” *J. Amer. Math. Soc.* 32 (2019), no. 3, 825–919.
  Constructs contact homology in general. This paper is very technical.

Equivariant 3-manifold topology

- *P. A. Smith, “Transformations of finite period.” *Ann. of Math.* (2) 39 (1938), no. 1, 127–164.
  and/or
  P. A. Smith, “Fixed-point theorems for periodic transformations.” *Amer. J. Math.* 63 (1941), 1–8.
  Give various restrictions on the fixed set of an action of Z/p on a

sphere, and formulate the Smith Conjecture. The first paper has the more famous results, but the second has similar ideas and is easier to read.

- **The Smith conjecture*. Papers presented at the symposium held at Columbia University, New York, 1979. Edited by John W. Morgan and Hyman Bass. Pure and Applied Mathematics, 112. Academic Press, Inc., Orlando, FL, 1984.

  First three chapters give background and an overview of the proof. The proof splits into various components, including minimal surfaces and Thurston's work on geometrization. Perhaps someone could present the first three chapters and someone else the Gordon-Litherland and Meeks-Yau chapters.
- W. Meeks and S. T. Yau, "The classical Plateau problem and the topology of three-dimensional manifolds. The embedding of the solution given by Douglas-Morrey and an analytic proof of Dehn's lemma." *Topology* 21 (1982), no. 4, 409–442.

  Uses minimal surfaces to prove an equivariant version of Dehn's lemma. Related to the proof of the Smith conjecture, and to Edmonds's work on periodic knots.
- A. Edmonds, "Least area Seifert surfaces and periodic knots." *Topology Appl.* 18 (1984), no. 2-3, 109–113.

  Uses minimal surfaces to give restrictions on which knots can be periodic (have rotational symmetries).
- K. Murasugi, "On periodic knots." *Comment. Math. Helv.* 46 (1971), 162–174.

  Gives restrictions on the Alexander polynomial of periodic knots (knots with rotational symmetries).
- K. Murasugi, "Jones polynomials of periodic links." *Pacific J. Math.* 131 (1988), no. 2, 319–329.

  Give analogous restrictions for the Jones polynomial.
- S. Henry and J. Weeks, "Symmetry groups of hyperbolic knots and links." *J. Knot Theory Ramifications* 1 (1992), no. 2, 185–201.

  Uses hyperbolic geometry to compute the symmetry group of almost any knot.
- S. Naik, "Equivariant concordance of knots in S^3". *KNOTS '96* (Tokyo), 81–89, World Sci. Publ., River Edge, NJ, 1997.

  Introduces the idea of equivariant concordance between knots, and gives some first examples of and restrictions on it.
- K. Boyle and J. Musyt, "Equivariant cobordisms between freely-periodic knots." arXiv:2111.10678 (preprint).

  Determines when two freely periodic knots (knots preserved by a free Z/p action on S^3) are related by a symmetric cobordism.
- +C. Livingston, *Knot Theory.* Carus Mathematical Monographs, 24. Mathematical Association of America, Washington, DC, 1993.

  Has a chapter about periodic knots, giving examples of both Edmonds's and Murasugi's restrictions.

Topological concordance

- R. Fox and J. Milnor, "Singularities of 2-spheres in 4-space and cobordism of knots." *Osaka J. Math* 3 (1966) 257-267.

  Proves the famous Fox-Milnor condition for slice knots and gives various applications.
- J. Levine, "Knot cobordism groups in codimension two." *Comment. Math. Helv.* **44** (1969), 229-244.

  Shows the algebraic concordance group is isomorphic to a large subgroup of $Z^\infty \oplus (Z/2)^\infty \oplus (Z/4)^\infty$.
- A Tristram, "Some cobordism invariants for links." *Proc. Camb. Phil. Soc.* 66 (1969), 251-264.

  Defines Levine-Tristram signatures, a large collection of concordance invariants.
- A. Casson and C. Gordon, "Cobordism of classical knots." *A la recherche de la Topologie perdue*, Progress in Mathematics, Volume 62, 1986.

  Defines Casson-Gordon invariants and shows that many algebraically slice knots are not topologically slice. Another good reference is Casson-Gordon, "On slice knots in dimension three."
- M. Freedman and F. Quinn, *Topology of 4-manifolds*, Chapter 11. Princeton University Press, 1990.

  Proves that Alexander polynomial 1 knots are slice (among other things). Another good reference is a paper of Garoufalidis-Teichner.
- M. Hedden, P. Kirk, and C. Livingston, "Non-slice linear combinations of algebraic knots." *JEMS* 14 (2012), no. 4, 1181-1208.

  A more recent application of Casson-Gordon invariants.
- +S. Behrens, B. Kalmar, M. H. Kim, M. Powell, and A. Ray. *The disc embedding theorem*. Oxford University Press, 2021.
- +Charles Livingston, "A survey of classical knot concordance".