MATH 432/532 SPRING 2017
HOMEWORK 6
DUE FEBRUARY 17, 2017.

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Shortened, because we're behind schedule.

Problems:

(1) Lee, Exercise 3.7.

(2) Lee, Exercise 3.17.

(3) Define $f\colon \mathbb{R} \to S^1$ by
$$f(t) = (\cos(t), \sin(t)).$$
Compute $df_p(\frac{\partial}{\partial t})$ for each $p \in \mathbb{R}$ with respect to the local coordinate systems from Lee, Example 1.31. Check that your answers agree, in a suitable sense, on at least one overlap between charts.

(4) Consider the map $F\colon S^3 \to S^2$ defined in Lee's Problem 2-3(c). Choose some smooth coordinate chart (U, ϕ) on S^3 and some smooth coordinate chart (V, ψ) on S^2 so that $F(U) \subset V$, and compute dF with respect to your chosen coordinates.

Note. If your solution isn't more clever than mine, you will have some excruciating partial derivatives to compute. It's fine to just give them names rather than computing them (but do write out explicitly what would need to be computed).

Challenge problems (required for Math 532, optional for 432):

(5) Define $f\colon \mathbb{R}P^2 \to \mathbb{R}^3$ by
$$f([x, y, z]) = \left(\frac{yz}{x^2 + y^2 + z^2}, \frac{xz}{x^2 + y^2 + z^2}, \frac{xy}{x^2 + y^2 + z^2}\right).$$
Show that there are exactly six points $p \in \mathbb{R}P^2$ at which df_p is not injective by computing df in coordinate charts from Lee, Example 1.5.

(This exercise is adapted from Spivak's *Comprehensive introduction to differential geometry*, Problem 2.29.)

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