(1) Following Ozsváth-Szabó following Turaev, we defined a spin\(^c\)-structure on \(Y\) as a homology class of non-vanishing vector fields. Prove that this agrees with the usual definition of spin\(^c\) structures, as a lift of the classifying map

\[
\begin{array}{ccc}
BSpin^c(3) & \to & Y \\
& \searrow & \downarrow \\
& & BSO(3)
\end{array}
\]

or, equivalently, a principal spin\(^c\)(3)-bundle \(P\) over \(Y\) and a bundle map

\[
\begin{array}{ccc}
P & \to & \text{Fr}(TY) \\
& \searrow & \\
& & Y
\end{array}
\]

interwining the actions of \(Spin^c(3)\) and \(SO(3)\) (via the projection \(Spin^c(3) \to SO(3)\)). (If you use the principal bundle route, flesh out the definition first.) (Hint: this takes some thought. For one direction, given a non-vanishing vector field, the orthogonal complement is a 2-plane bundle, which gives a lift to \(BSO(2) = BS^1\).)

(2) Prove that the set of spin\(^c\)-structures on \(Y\) is an affine copy of \(H^2(Y)\). Also, given a spin\(^c\)-structure there is an induced complex line bundle, coming from the quotient map \(Spin^c(n) \to U(1)\). This line bundle has a first Chern class. How does this first Chern class relate to the \(H^2(Y)\)-action on the set of spin\(^c\)-structures?

(3) Prove directly that if \(m(K)\) is the mirror of \(K\) then \(\tau(m(K)) = -\tau(K)\).

(4) Compute \(CFK^-(S^3, K)\) and \(\tau(K)\) for some more knots represented by genus 1 Heegaard diagrams (e.g., the figure 8 knot).

(5) Use a genus 1 Heegaard diagram to compute \(\tau(K)\) for all torus knots \(T_{p,q}\). (You should get \((p - 1)(q - 1)/2\). \textit{Warning.} I don’t know how to do the computation this way.)

(6) Find an unknotted of \(T_{p,q}\) using \((p - 1)(q - 1)/2\) crossing changes.

(7) Prove that the Euler characteristic of \(\widehat{HFK}\) is the Alexander polynomial as follows. Fix a knot diagram for a knot \(K\) and let \(H\) be the associated Heegaard diagram. Show that the generators for \(\widehat{CFK}(H)\) correspond to the Kauffman states (in the sense of Louis Kauffman, \textit{Formal knot theory}). Study how the gradings behave under this identification. (This is involved; start with some examples. See Ozsváth-Szabó, \textit{"{H}eegaard Floer homology and alternating knots"}, for details. There are other proofs.)
(8) Use a genus 1 Heegaard diagram to compute $CFK^\infty$ of the figure 8 knot, and then use the large surgeries formula to compute $\widehat{HF}$ and $HF^+$ for large surgeries on the figure 8 knot.

(9) The simplest version of the surgery exact triangle is an exact sequence

$$\cdots \to HF^+(S^3) \to HF^+(S^3_n(K)) \to HF^+(S^3_{n+1}(K)) \to HF^+(S^3) \to \cdots.$$ 

Deduce that the surgery exact triangle holds for large $n$ surgeries on a knot $K$ in $S^3$ from the large surgery formula. As a corollary, if some large surgery on $K$ is an $L$-space then all larger surgeries on $K$ are $L$-spaces, as well.

(10) Prove that the differential on $CFK^\infty$ preserves the property that $A(x) + i - j = 0$.

(11) Prove that the intersection sign of a point in $T_\alpha \cap T_\beta$ gives a $\mathbb{Z}/2$-grading on $\widehat{CF}(\Sigma, T_\alpha, T_\beta)$. (This grading depends on the orientations of $T_\alpha$ and $T_\beta$, and hence is not an invariant. It is an invariant as a relative $\mathbb{Z}/2$-grading, and can be lifted to a well-defined absolute $\mathbb{Z}/2$-grading. What I’m asking you to prove is just that the differential on $\widehat{CF}$ changes this grading by 1.)

(12) Deduce a large $n$ surgery formula for $\widehat{HF}(S^3_n(K))$ from the surgery formula for $HF^+$ and the exact sequence relating $HF^+$ and $\widehat{HF}$.

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