MATH 635 HOMEWORK 1
DUE JANUARY 16, 2019.

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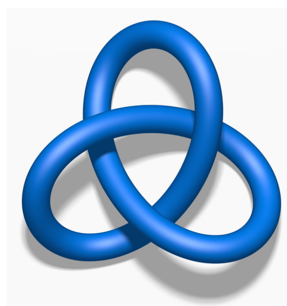
Reminder: homework is due at the *beginning* of class, handed to me or in my mailbox on the second floor of Fenton.

(1) Hatcher, 2.2.7, p. 155.
(2) Hatcher, 2.2.9(a,b,c), p. 156.
(3) The *solid torus* is the space $S^1 \times D^2$; this is a donut.
   (a) Use cellular homology to compute the homology of the solid torus.
   (b) Use the Mayer-Vietoris sequence to compute the homology of the solid torus.
(4) Let K be the trefoil knot:



   (a) Use cellular homology to prove that $H_1(S^3 \setminus \mathrm{nbd}(K)) \cong \mathbb{Z}$ and $H_2(S^3 \setminus \mathrm{nbd}(K))$ is trivial. (Hint: remember Hatcher exercise 22, p. 55.)
   (b) Given a map $\gamma\colon S^1 \to S^3 \setminus K$, the *linking number* of γ with K is the degree of the map
$$\mathbb{Z} = H_1(S^1) \to H_1(S^3 \setminus K) \cong \mathbb{Z}.$$
   (This is well-defined up to sign.)
   Let $\phi\colon \partial(S^1 \times D^2) = S^1 \times S^1 \to \partial\, \mathrm{nbd}(K)$ be a homeomorphism. Compute the homology of
$$S^3_\phi(K) := (S^3 \setminus \mathrm{nbd}(K)) \cup_\phi (S^1 \times D^2)$$
   in terms of the linking number of $\phi(\{pt\} \times S^1)$ and K. (The manifold $S^3_\phi(K)$ is called a *surgery on* K.) (Hint: you can solve this either using cellular homology or the Mayer-Vietoris theorem.)
(5) Define the cellular cochain complex and prove that cellular cohomology is isomorphic to singular cohomology.
(6) Hatcher, 2.2.33, p. 158.

Review / qualifying exam practice (not to turn in):

(1) Hatcher 2.2.2, 2.2.3, 2.2.4, 2.2.14, 2.2.15, 2.2.19, 2.2.28, 2.2.29, 2.2.30
(2) With notation as in Problem 4, give two non-homotopic maps γ with linking number ± 1 with K. (Hint: think about π_1.)

More problems to think about but not turn in:

(1) Hatcher, 2.2.8, p. 155. Also, deduce the fundamental theorem of algebra.
(2) Give an example of a surgery on the trefoil which is has trivial H_1 but is not homeomorphic to S^3.

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