MATH 635 HOMEWORK 1
DUE JANUARY 12, 2021.

INSTRUCTOR: ROBERT LIPSHITZ

Updated January 5: added some words about Problem 7.

(1) Hatcher 2.1.26 (p. 133)
(2) Hatcher 2.1.27 (p. 133)
(3) The *solid torus* is the space $S^1 \times D^2$; this is a donut. Use the Mayer-Vietoris sequence to compute the homology of the solid torus.
(4) Let K be a knot in S^3, that is, a smoothly embedded circle. It follows from the implicit function theorem that K has a neighborhood U homeomorphic to $S^1 \times D^2$, so that K is identified with $S^1 \times 0$. Use the Mayer-Vietoris sequence to compute $H_1(S^3 \setminus K)$. (Hint: cover S^3 by U and $S^3 \setminus K$.)
(5) Hatcher 2.2.28 (p. 157)
(6) Hatcher 2.2.33 (p. 158)
(7) Hatcher 2.2.34 (p. 158): use the Mayer-Vietoris sequence to prove the long exact sequence for a pair. (You can prove the long exact sequence in reduced homology if you prefer.) Then explain why Hatcher removed this problem in the updated edition of his textbook, i.e., why he doesn't like your solution.

Suggested review / qualifying exam practice (not to turn in):

(1) Hatcher 2.1.29, 2.1.30, 2.1.31.
(2) Hatcher 2.2.29, 2.2.30, 2.2.31, 2.2.32, 2.2.35, 2.2.36.

Email address: lipshitz@uoregon.edu