

SYLLABUS for MATH 256

I] First Order ODEs: Preliminaries

- A) Initial Value Problem
- B) Graphs
(No crossing)

II] Finding Explicit Solutions of Certain First Order ODEs

- A) Basic Integrable
- B) Separable
- C) Linear First Order
- D) First Order Homogeneous

III] Story Problems and Applications

- A) Mixing
- B) Populations

IV] What to Do When You Can't Solve a First Order ODE

- A) Numerical Treatment
- B) Direction Fields and Graph Sketches
- C) Phase Portraits
(1 Dimensional)
- D) Equilibria and Stability
(Compare with Phase Portraits)

V] Second Order ODEs: Preliminaries

- A) Initial Value Problem
- B) Graphs
(Crossing allowed)

VI] Second Order ODEs That Reduce to First Order

- A) Missing Dependent Variable
- B) Missing Independent Variable

VII] Linear Second Order ODEs: Preliminaries

- A) Existence and Uniqueness of Solutions for the Initial Value Problem
- B) Vector Spaces and Linear Operators
 - 1) Theory of vector spaces
(linear independence, bases)
 - 2) Theory of linear operators and their kernels
- C) Applying Vector Space Theory to Linear ODEs
 - 1) Superposition of Solutions
 - 2) Homogeneous and Nonhomogeneous Linear ODEs
 - 3) General Solutions of Linear ODEs
- D) Finding the General Solution of a Second Order Linear Homogeneous ODE When You Know One Solution

VIII] Solving Second Order Linear ODEs with Constant Coefficients

- A) Homogeneous
Use $h(t) = e^{sp(kt)}$ ansatz
 - 1) 2 real roots

- 2) Double real root
- 3) Complex roots
- B) Story Problems
 - (Oscillators, circuits)
- C) Inhomogeneous
 - 1) $y(t) = h(t) + p(t)$
 - 2) Finding $p(t)$
- D) Comments on Higher Order Linear ODEs with Constant Coefficients

IX] Linear First Order Systems

- A) Preliminaries
 - 1) What They Look Like
 - 2) Where They Come From
 - a> Interacting populations
 - b> First Orderizing Higher Order ODEs
 - 3) Initial Value Problem
- B) Model Story Problems
- C) Explicit Solutions for Semi-Decoupled First Order Systems
- D) Homogeneous Constant Coefficient Linear First Order Systems
 - 1) Explicit Solutions
 - a> $(X(t), y(t)) = (v, w) \exp(rt)$ Ansatz
 - > Eigenstuff problem
 - b> The eigenvalues and eigenvectors and the solutions
 - 2) Qualitative analysis
 - a> Direction fields on phase portraits
 - (i) sketching
 - (ii) the zoo of phase portraits
 - (classified by eigenstuff)
 - b> Equilibria and stability