

Calculus III - Math 253
Syllabus 2016-2017
Course coordinator: Dan Dugger

Text: *Calculus, Concepts and Contexts, 4th edition*, Stewart. We will cover Chapter 8 together with power series solutions to differential equations.

Chapter 8 should be thought of as building to Taylor Polynomials and Taylor's Remainder Theorem, and also to power series representation of functions. Along the way, one considers sequences and sequence convergence, series and series convergence.

Finally, as an additional application of power series, we'd like to cover power series solutions to differential equations. This isn't covered by the text, but should only be covered in the most elementary way considering differential equations of the form

$$y' + p(x)y = q(x) \text{ and } y'' + p(x)y' + q(x)y = r(x)$$

for p, q, r polynomials, and thus staying away from discussions of things like ordinary points, singular points, regular singular points, etc. Appendix B from Blanchard, Devaney and Hall's ODE book could be used to cover this (or the relevant chapter from any basic textbook on differential equations). It includes homework problems.

Course Goals: The primary goal of the course is to bring students to a point where they can **use Taylor's theorem in a reasonably effective way**; at least on standard Taylor polynomial approximations like those for $\sin(x)$, $\cos(x)$, e^x and $\log(x)$.

This means they need to be able to compute the Taylor polynomials, and then (this is the difficult part) *use Taylor's theorem to estimate the error!* The remainder theorem appears in 8.7, and applications of the remainder theorem are section 8.8. Note that this comes well into the term, and you want be sure and reach this point when there are enough weeks left in the term to give students practice doing the sorts of exercises that occur in 8.8 before the final.

Here is a list of course goals that includes some less central points

- Show sequences *don't* converge by using ϵ - N definition of limit.
- Use standard series convergence tests.
- Estimate sums using the integral test when possible, the alternating series test when possible and the comparison test when possible. See section 8.3, 8.4.
- Calculate radii of convergence for a power series, calculate Taylor series, represent common transcendental functions as power series.
- **Use Taylor's remainder theorem to approximate values of transcendental functions to given levels of accuracy.**
- Give power series solutions to appropriate differential equations. Recognize solutions when they are common transcendental functions.

Note that the course goals above emphasize applications and this is what the course should do in general. But it is appropriate to introduce the precise definition of limit of a sequence (it is in Appendix D) and explain that one needs this definition to prove various facts that will be stated without proof in the course. In addition, one could then use that definition to prove a very small number of elementary things. For example, one could prove that a

given sequence can't have two different limits, and one could use the definition to prove that certain sequences don't have limits.

Homework: It is hard to use WebWork as the primary homework tool in this class, as most problems involve deciding whether or not a certain sequence/series converges and justifying the answer. A few problems on WebWork might be used for supplementary purposes, but it's not clear if this is worth it. See

<http://pages.uoregon.edu/ddugger/ma253.html>

for the homework assignments I last used.

Warnings: This is a difficult class to teach. The applications of the material to science students are not as readily accessible as they were in 251-252, and the problems are not just about "getting an answer" anymore. Understanding the subtleties of convergence requires a logical and mathematical framework that most students are encountering for the first time, and they need a lot of help with this. It possibly makes sense to develop more training material along these lines, to be included in the course, but we are not there yet.

Here are some specific comments:

- (1) You cannot assume that students know what factorials are, or what binomial coefficients are. Make sure to spend adequate time explaining these things.
- (2) Students have no idea what is bigger than what. It is useful to spend some time going over why

$$\text{constant} < \log x < x^{0.1} < x < x^{1.1} < x^2 < 2^x < 3^x < x! < x^x$$

(where for some of these inequalities I am being sloppy about the applicable values of x).

- (3) Learning precise mathematical statements of definitions and theorems is a key part of this course. Don't expect students to realize that they should do this, or even to know *how* to do it. It is best to be clear about your expectations here, and to give the students practice with this before assessing them on an exam.
- (4) There's a certain sense to starting the class with Taylor polynomials and with the whole idea of approximating a function by a polynomial. You can even talk about approximate solutions of ODEs in a naive sense. At some point this naturally leads into the idea of infinite series, and that brings you to the convergence material in a way that highly motivates it. Note that this approach allows students to work with Taylor's formula for the whole course, rather than just restricting it to the last half. Unfortunately, the textbook is not written this way... I have taught the course in this manner and felt like it worked well, though. See the second schedule below.

APPROXIMATE SCHEDULE (CLASSICAL APPROACH)

Week 1	8.1, A32-33	Week 6	8.7
Week 2	8.2, 8.3	Week 7	8.7, 8.8
Week 3	8.3, 8.4	Week 8	8.8, review (exam 2)
Week 4	8.4, 8.5 (exam 1).	Week 9	Power series and DEs
Week 5	8.5, 8.6	Week 10	Review, catch up.

APPROXIMATE SCHEDULE (ALTERNATIVE APPROACH)

Week 1	8.7	Week 6	8.6
Week 2	8.1, A32–33	Week 7	8.7
Week 3	8.2, 8.3	Week 8	8.7, review (exam 2)
Week 4	8.4 (exam 1).	Week 9	8.8 and DEs
Week 5	8.4, 8.5	Week 10	Review, catch up.

Technology: This is a good course to incorporate modern technology, if you feel so inclined. It is useful for students to learn how to use Mathematica to crank out Taylor approximations, compute finite sums of series, and other tedious tasks. Mathematica can also be a good tool for exploring questions around convergence and around Taylor’s Theorem. For some homework questions devoted to this kind of thing, see my webpage from the last time I taught the course (cited in the Homework section above). What we have now is only a beginning, and it would be nice to develop homework problems that use the technology in more interesting ways. This stuff is also available (with TeX files) in the department’s “calculus materials depository”.