

Due Wednesday November 19th.

5.9, 5.16, 5.18, 6.1

And the following

Consider $(0, 1)$ as a 1-manifold. Suppose I define a measure on $(0, 1)$ on an open set $A \subset (0, 1)$ by

$$\mu_1(A) = \int_A f_1(x) dx$$

for a fixed smooth function f (for now, you can assume that A is a collection of open intervals - don't worry about σ -algebras). Now consider a diffeomorphism

$$F : (0, 1) \rightarrow (0, 1)$$

and define a measure on $(0, 1)$ by

$$\mu_2(B) = \mu_1(F^{-1}(B)).$$

There is a function f_2 (you can assume this fact) such that

$$d\mu_2(B) = \int_B f_2(x) dx.$$

Show by counterexample that the following statement fails:

$$f_1 \in L^\infty(dx) \implies f_2 \in L^\infty(dx)$$