Exam 1 Review (Sections 7.1 – 7.3)

CQ #1: Consider the table of values for functions $x(t)$, $y(t)$, and $z = f(x, y)$. Compute $\frac{dz}{dt}(2) = z'(2)$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x(t)$</th>
<th>$y(t)$</th>
<th>$x'(t)$</th>
<th>$y'(t)$</th>
<th>$f_x(x, y)$</th>
<th>$f_y(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>7</td>
<td>-2</td>
<td>3</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>9</td>
<td>12</td>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>-4</td>
<td>6</td>
</tr>
</tbody>
</table>

a) $-19$
b) $21$
c) $-10$
d) $1$
e) $-3$

\[
z(t) = f(x(t), y(t))
\]
\[
z'(t) = f_x(x(t), y(t))x'(t) + f_y(x(t), y(t))y'(t).
\]

\[
z'(t) = (-4)(7) + (6)(3)
\]
\[
= -28 + 18 = -10.
\]

$\boxed{C}$
CQ #2: The quantity sold of a commodity is \( q(x, p) \) where \( x \) is the unit price of the commodity sold and \( p \) is the price of a similar commodity. The function \( s(x, p) \) gives the demand for the similar commodity. Write a formula for the profit from production and sale of both products, assuming there is a cost of $100 to produce each unit of each commodity.

a) \( q(x, p) + s(x, p) \)
b) \( x \cdot q(x, p) + p \cdot s(x, p) \)
c) \( p \cdot q(x, p) + x \cdot s(x, p) \)
d) \( p \cdot q(x, p) + x \cdot s(x, p) - 100(x + p) \)
e) \( x \cdot q(x, p) + p \cdot s(x, p) - 100[q(x, p) + s(x, p)] \)

\[
\text{Profit} = \left( \text{Revenue} - \text{Cost} \right) \text{ for first commodity} + \left[ \text{Revenue} - \text{Cost} \right] \text{ for second commodity.}
\]

\[
\text{Price} = x \text{ and } p.
\]

\[
x \cdot q(x, p) - 100(q(x, p)) + p \cdot s(x, p) - 100 \cdot s(x, p)
\]
CQ #4: Let $S(x, y)$ be the current market share (as a fraction) of Wang’s Widgets Inc., when the current market share of its chief rival Wu’s Widgets Inc. is $x$, and $y$ is Wang’s Widgets Inc.’s market share from the previous year. Interpret the expression $S_x(x, y)$ in the context of the model.

a) The current value of Wang’s Widgets Inc.’s market share.
b) The rate of change in Wang’s Widgets Inc.’s market share, measured in market share per market share of Wu’s Widgets Inc.
c) The rate of change in Wu’s Widgets Inc.’s market share, measured in current market share per market share of Wang’s Widgets Inc. last year.
d) The rate of change in Wu’s Widgets Inc.’s market share, measured in market share per market share of Wang’s Widgets Inc.
e) The rate of change in Wang’s Widgets Inc.’s market share, measured in market share per market share of Wu’s Widgets Inc. last year.
CQ #3: The quantity sold of a commodity is $q(x, p)$, where $x$ is the unit price of the commodity sold and $p$ is the price of a similar commodity. The function $s(x, p)$ gives the quantity sold of the similar commodity. If we expect the two commodities to be complementary, which of the following must be true?

a) $q(x, p) < 0$ and $s(x, q) < 0$.
b) $q_x(x, p) < 0$ and $s_p(x, p) < 0$.
c) $q_p(x, p) < 0$ and $s_x(x, p) < 0$.
d) $q_x(x, p) > 0$ and $s_p(x, p) > 0$.
e) $q_p(x, p) > 0$ and $s_x(x, p) > 0$.

If the price of one is sold, expect more of the other to be sold.

[not usually the case with similar commodities]

If price of one goes up, expect its sales to go down, so expect the sales of the other to go down.

Price of first increases $\Rightarrow$ demand to second goes down.

And $q_x(x, p) < 0$.

This is $\boxed{c}$. 
CQ #5: Compute $f_{nv}(1, 2)$ where $f(n, v) = 3n^2v + \ln(nv)$.

a) 6  
b) 13  
c) 5.75  
d) 8  
e) 6.7

\[
\begin{align*}
\frac{\partial}{\partial v} (6nv + \frac{1}{nv}) & = 6n + 0 = 6n, \\
\frac{\partial}{\partial n} (6nv + \frac{1}{nv}) & = 6n + 0 = 6n.
\end{align*}
\]

Put $n=1$ and $v=2$ to get $(6)(1) = 6.$
CQ #6: Describe the domain of the function $f(x, y) = \sqrt{3x^2 - y}$ in the $xy$-plane.

a) All points in the $xy$-plane.
b) All points on the curve $y = 3x^2$.
c) All points on or above the curve $y = 3x^2$.
d) All points on or below the curve $y = 3x^2$.
e) All points not on the curve $y = 3x^2$.

We need $3x^2 - y \geq 0$ so its square root is real.

So $3x^2 \geq y$

So $y \leq 3x^2$

$y = 3x^2$ is the curve which appears in most of the answers.

$y \leq 3x^2$ says:

on or below the curve.

\[ \text{The black and red points are in the domain. The green point is not in the domain.} \]