Instructions:

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. The exam is worth 33 points. That means you should spend around 1.5 minutes for each point the problem is worth in order to complete the exam in time.

Partial credit is possible. Any fill blank or multiple choice items with space left for “work shown (partial credit possible)” can receive up to half credit for the work shown. Partial credit is always available on free response questions. In the limited space provided, be careful to only include what you want your instructors to evaluate.

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Bonus Codes:
Multiple Choice and Fill Blank Choose the best answer from among the multiple choices given. In each answer blank, write the correct numerical, symbolic expression or phrase (e.g., “DNE”). Numerical answers can be expressed exactly or rounded to three decimal places.

1. Let \( z = N(x, y) = e^{-4xy^2} \).

   (a) Fill Blank: Compute \( N_y \).

   \[
   N_y = \frac{-8y e^{-4xy^2} + 32xy^3 e^{-4xy^2}}{4xy^2}
   \]

   (b) Multiple Choice: Sketch the graph of the trace \( y = -2 \) for \( N \).

   \[
   y = -2 : e^{-4x \cdot (-2)}^2 = e^{-10x}
   \]

2. Fill Blank: Let \( z = f(x, y) = \ln(4x^2 - 5y^2) \) and assume additionally that \( x = 2 - 7t - t^2 \), while \( y = 3t \) for some variable \( t \). Compute \( \frac{dz}{dt} \). Your answer may contain \( x, y, \) and \( t \).

   \[
   \frac{dz}{dt} = \frac{8x}{4x^2 - 5y^2} \cdot (-7 - 2t) + \frac{10y}{4x^2 - 5y^2} \cdot 3
   \]

3. Fill Blank: Write the three equations required to find the maximum value of \( f(x, y) = 2y^3 - x^2 y^2 \) subject to the constraint \( 4y - 3x = 10 \), according to the method of Lagrange multipliers.

   Equation: __________________________

   Equation: __________________________

   Equation: __________________________

4. Multiple Choice: Consider the function \( z = f(x, y) = \frac{8y^2 - 5y}{y - 4x} \). The domain of \( f \) is all points ...

   (a) ... in the \( xy \)-plane.

   (b) ... in the \( xy \)-plane not on \( x = 0 \) nor \( y = 0 \).

   (c) ... in the \( xy \)-plane not on \( x = 1.6y^2 \).

   (d) ... in the \( xy \)-plane above \( y = 4x \).

   (e) ... in the \( xy \)-plane on \( y = 4x \).

   (f) ... in the \( xy \)-plane not on \( y = 4x \).

   \[
   \text{Need: } y - 4x \neq 0 \quad \Rightarrow \quad y \neq 4x
   \]
5. Fill Blank: Consider the savings function $S(x,y)$ dollars saved when an individual has an annual income of $x$ and their spouse has an annual income of $y$ (each in thousands of dollars). Currently, the two people together make $410,000$ and have a maximum savings when the Lagrange multiplier is equal to $-0.1$. By approximately how much would the maximum savings change if the two people made $3$ thousand dollars more annually?

Work shown (partial credit possible)

6. Multiple Choice: Suppose that for a twice-differentiable function $f(x,y)$ we know that $f_{xx}(5,9) = 8$ and $f_{yy}(5,9) = 4.5$ and $f_{xy}(5,9) > 0$. Then $f$ will have a saddle point at $(5,9)$ as long as ...

(a) $f_{xx}(5,9) > 36$.
(b) $f_{xx}(5,9) > 6$.
(c) $f_{xx}(5,9) > 49$.
(d) nothing else, $f_{xx}(5,9) > 0$ is sufficient.
(e) nothing, $f$ cannot have a saddle point at $(5,9)$.

Work shown (partial credit possible)

$$f_{xx} \cdot f_{yy} - (f_{xy})^2 < 0$$

$$g(4.5) - (f_{xy})^2 < 0$$

$$36 - (f_{xy})^2 < 0$$

$$f_{xy} > \sqrt{36} = 6$$

7. Multiple Choice and Fill Blank: Let $t$ be the time (in weeks) after the beginning of the year and $x$ be the annual demand (in thousands of boxes) for a product being kept in a warehouse. The remaining inventory in the warehouse is given by $I(x,t) = xe^{-0.2t}$ thousand boxes. Construct a sentence interpreting the value of $I_x(4,10)$ in the applied context using the four parts below.

(a) Choose the first part.

i. The annual demand is...
ii. The time is...
iii. The remaining inventory is...

(b) Choose the second part.

i. changing at this rate...
ii. maximized at this value...
iii. minimized at this value...
iv. equal to this value...

(c) Write numbers for the third part.

"... after _____ weeks,
and with annual demand of
4000 boxes ...
"

(d) Choose the final part.

i. while demand changes and inventory remains constant.
ii. while demand changes and time remains constant.
iii. while time changes and inventory remains constant.
iv. while time changes and demand remains constant.
Free Response Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written either in exact (unsimplified) or in approximate form as long as an exact solving method is used. Clearly mark your final answer, and include units in all relevant parts.

8. One company makes two products, where the price of the first product is \( n \) (in rupees) and the price of a second product is \( p \) (also in rupees). Monthly demand for the first product is \( f(n, p) = 50 - 6n + 4p \) items while monthly demand for the second product is \( g(n, p) = 80 + 5n - 6p \) items.

(a) Are the two products substitutes, complementary, or neither? Show work to justify your answer.

\[
\begin{align*}
\text{Need: } f_p &= 4 \\
& \quad \text{g}_n = 5 \\
\{ & \text{Both are } >0, \text{ so the product are substitutes}.
\end{align*}
\]

(b) What prices maximize the total revenue from the production and sale of both of these items? Use calculus to verify that the value is a maximum.

\[
R = p_1 \cdot q_1 + p_2 \cdot q_2 = n (50 - 6n + 4p) + p (80 + 5n - 6p)
\]

\[
= 50n - 6n^2 + 9np + 80p - 6p^2
\]

\[
\begin{align*}
R_n &= 50 - 12n + 9p = 0 & \Rightarrow & n = \frac{50 - 9p}{12} \\
R_p &= 9n + 80 - 12p = 0 \\
q \left( \frac{50}{12} + \frac{9}{12}p \right) + 80 - 12p = 0 \\
37.5 + 6.75p + 80 - 12p &= 0 \\
-5.25p &= -117.5 \\
p &\approx 22.38
\end{align*}
\]

\[
n = \frac{50}{12} + \frac{9}{12}p \approx \frac{50}{12} + \frac{9}{12} (22.38) \approx 20.95 \quad (20.95, 22.38)
\]

\[
\begin{align*}
R_{nn} &= -12 \\
R_{pp} &= -12 \\
R_{np} &= 9
\end{align*}
\]

\[
D(20.95, 22.38) = R_{nn} \cdot R_{pp} - (R_{np})^2 = (-12) \cdot (-12) - (9)^2 = 63 > 0
\]

and \( R_{nn} < 0 \), so a price of 20.95 rupees for the first and 22.38 rupees for the second gives max \( R \).