**Exam 1**

Math 242, Winter '18vb

Your Name: _______________ Solutions _______________ Student Number: _______________

Your Discussion Leader's Name (circle one):

- Dana Fry
- Mike Gartner
- Eli Hultbert

Your Discussion Day and Time (circle one):

- Wed 4pm
- Wed 5pm
- Thurs 9am
- Thurs 10am

Instructions:

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. The exam is worth 33 points. That means you should spend around 1.5 minutes for each point the problem is worth in order to complete the exam in time.

Try for partial credit. Any fill blank or multiple choice items with space left for "work shown (partial credit possible)" can receive up to half credit for the work shown. Partial credit is always available on free response questions. In the limited space provided, be careful to only include what you want your instructors to evaluate.

Be ethical. Provide your own work on the exam, without cheating. Being found responsible for academic misconduct, including communicating with others about exam content before the exams have been returned, is considered cheating and will result in a score of zero on this exam. Sign below to indicate you understand.

Your Signature (required for grading of exam): __________________________

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<tr>
<td>Points Possible</td>
<td>11</td>
<td>10</td>
<td>12</td>
<td>33</td>
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</tbody>
</table>

**Bonus Codes:**


Multiple Choice and Fill Blank Choose the best answer from among the multiple choices given. In each answer blank, write the correct numerical, symbolic expression or phrase (e.g., “DNE”). Numerical answers can be expressed exactly or rounded to three decimal places.

1. Let the total consumption (spending, in thousands of European euro) by a consumer having \( n \) thousand euro of income \( t \) years from now be given by

\[
C(n, t) = \frac{n^2 + 3n}{n + 9} \cdot e^{-0.1t}.
\]

(a) Fill Blank: Compute \( C(7, 4) = \) ___________

\[
\boxed{2.933}
\]

(b) Fill Blank: Compute \( C_n(7, 4) = \) ___________

\[
\boxed{0.529}
\]

(c) Multiple Choice: The units on the answer from part (b) are

i. thousand euro per year

ii. thousand euro

iii. years

iv. years per thousand euro

v. thousand euro per thousand euro

2. Consider the function \( z = f(s, t) = -3tse^{-t^2+5} \).

(a) Identify the graph of the trace \( t = 2 \) in the \( sz \)-plane.

\[\boxed{(ii)}\]

(b) Compute the mixed partial derivative \( f_{st} = \) ___________

\[
\boxed{-3e^{-t^2+5} + 6te^{-t^2+5}}
\]
3. Two competing companies sell subscriptions to similar music apps, Songdorn and Phyify. A demand model for Songdorn is \( S(a, b) = 200 - 8a + 2b \) where \( a \) is the monthly subscription price for Songdorn and \( b \) is the monthly subscription price for Phyify. Similarly, a demand model for Phyify is \( P(a, b) = 300 + 5a - 12b \).

\[ R = a \cdot S(a, b) = a \cdot (200 - 8a + 2b) \]

(a) **Multiple Choice:** Write a formula for the revenue obtained just from subscriptions to Songdorn.

- i. \( R(a, b) = 300a + 5a^2 - 12ab \)
- ii. \( R(a, b) = 200b - 8ab + 2b^2 \)
- iii. \( R(a, b) = 500a - 3a^2 - 10ab \)
- iv. \( R(a, b) = 300b + 5ab - 12b^2 \)
- v. \( R(a, b) = 200a - 8a^2 + 2ab \)
- vi. \( R(a, b) = 500b - 3ab - 10b^2 \)

(b) **Fill Blank:** Compute the first partial derivatives of the demand functions:

- \( S_a = -8 \)
- \( S_b = 2 \)
- \( P_a = 5 \)
- \( f_y = -12 \)

(c) **Multiple Choice:** Circle the two expressions from part (b) which tell us that the products are substitutes.

- Substitutes \( \iff \) \( S_b > 0 \) and \( P_a > 0 \)

4. **Fill Blank:** Consider the table of values for \( x \) and \( y \) each as functions of \( t \). Additionally, for \( z = f(x, y) \), we have \( f_x = 3xy \) and \( f_y = xy^2 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x(t) )</th>
<th>( y(t) )</th>
<th>( x'(t) )</th>
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<tr>
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<td>4</td>
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<td>-5</td>
<td>3</td>
</tr>
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Compute \( \frac{dz}{dt} \) at \( t = 2 \) using the chain rule:

\[ \frac{dz}{dt} \bigg|_{t=2} = \text{100} \]

5. **Multiple Choice:** Let \( g(p, q) = \ln(q - 9 + p^2) \). Describe its domain as a region in the \( pq \)-plane.

- (a) The region below the graph of \( q = 9 + p^2 \)
- (b) The region above the graph of \( q = 9 - p^2 \)
- (c) The region below the graph of \( q = 9 - p^2 \)
- (d) The region above the graph of \( q = 9 + p^2 \)
- (e) The region on the graph of \( q = 9 + p^2 \)

\[ g(p, q) = \ln(q - 9 + p^2) \]

Work shown (partial credit possible)

\[ \frac{dz}{dt} = f_x \cdot \frac{dx}{dt} + f_y \cdot \frac{dy}{dt} \]

\[ = (3xy) \cdot x' + (xy^2) \cdot y' \]

\[ \frac{dz}{dt} \bigg|_{t=2} = (3 \cdot 5 \cdot 4 \cdot (-1)) + (5 \cdot 4^2) \cdot (2) \]

Work shown (partial credit possible)

Need: \( q - q + p^2 > 0 \)

\[ q > q - p^2 \]
Free Response Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written either in exact (unsimplified) or in approximate form as long as an exact solving method is used. Clearly mark your final answers, and include units in all relevant parts.

6. The total cost to bring a new drug to market is determined to be \( T(x, y) = \frac{48}{x} + \frac{36}{y} + xy \) million dollars, when \( x \) thousand worker-hours of research and \( y \) thousand worker-hours of development are put into the product. Use the space below to answer the questions: (a) At what levels of research and development is total cost minimized? (b) What is that minimum cost? (c) Use calculus to verify that the value you found corresponds to a relative minimum of \( T \).

(a) Find critical points:

\[
\begin{align*}
T_x &= -48x^{-2} + y = 0 \quad \Rightarrow \quad y = \frac{48x^{-2}}{
T_y &= -36y^{-2} + x = 0
\end{align*}
\]

\[-36(48x^{-2})^{-2} + x = 0
-36 \cdot \frac{1}{2304x^4} + x = 0
\]

\[x : \left(-\frac{1}{64} \cdot x^3 + 1\right) = 0
x = 0 \quad \text{or} \quad x^3 = -1/-64 \quad \Rightarrow \quad x = 4
\]

\[x = 0 : \text{y is undefined}
\]

\[x = 4 : \quad y = 48(4)^{-2} = 3 \]

\[(4, 3) \quad \text{is the only critical point, and so probably the min!} \]

(b) \( T(4,3) = \frac{48}{4} + \frac{36}{3} + (4)(3) = 36 \text{ million dollars} \)

(c) \( T_{xx} = 96x^{-3} \), \( T_{yy} = 72y^{-3} \), \( T_{xy} = 1
\]

\[D(4,3) = \left(96 \cdot 4^{-3}\right) \cdot \left(72 \cdot 3^{-3}\right) - (1)^2 = 3
\]

\(D > 0 \text{ and } f_{xx} > 0 \), so \((4,3)\) is the location of a local minimum (confirmed).