Guide for Section 5.2: Integration by Substitution

Math 242, Winter 2020

**Ex 1** Compute \( \int (3x + 1)^2 \, dx \) by first expanding the integrand.

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**Thm** (Integration by Substitution)
Choose a substitution and replace all variables (including the differential, for example \( dx \)) with the substituted variable. Make this substitution choice so that the integrand is easier to antidifferentiate.

**Ex 2** Redo the integral calculation from Example 1 using the substitution \( u = 3x + 1 \).
Ex 3 Consider the indefinite integral $\int x \sqrt{x^2 + 1} \, dx$.

a) Try to find the value of the integral using the substitution $u = \sqrt{x^2 + 1}$.

b) Try to find the value of the integral using the substitution $u = x^2 + 1$. 
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**Ex 4** Evaluate the indefinite integral: \[ \int \frac{x - 3}{x^2 - 6x + 3} \, dx \]

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**CQ** (Section 5.2, #2)

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**Ex 5** Try (and fail) to evaluate the indefinite integral: \[ \int e^{-t^2} \, dt \]
Ex 6 Exploring a new ‘dynamic pricing’ scheme, a company determines that the rate of change in monthly demand for a product is given by \( \frac{dq}{dp} = -12pe^{-0.5p^2} \), where \( q \) is the quantity (thousands of units) sold and \( p \) is the unit price (in dollars per unit). Find a function to represent the total number of units sold each month at a unit price of \( p \) dollars if the potential market (number of units sold if the product were free) is 12 thousand units.
Ex 7 A corporation’s profit, generated from advertising deals, is deposited continuously at a rate of 1.8 million dollars per year into an account (initially containing no money) which earns interest at an annual rate of 5%, compounded continuously.

a) Write a differential equation to model the change in value of the account over time.

b) Solve the initial value problem from part (7a).

CQ (Section 5.2, #3)
The number $S$ of shares available for the public to trade (a company’s “float”) at time $t$ hours after its initial public offering may follow a Gompertz model, which is of the form

$$\frac{dS}{dt} = -0.05S \cdot \ln \left( \frac{K}{S} \right)$$

for a positive constant $K$ which represents the number of shares in total. If there are 3 million shares in total, initially with float 2.8 million shares, find a model for the company’s float as a function of $t$. 