Exam 2 (Sections 5.1 – 5.3) Math 242, Winter ’19va

Your Name: Solutions

Student Number: 

Discussion Leader’s Name (circle one): Yang Hu  Eli Hulbert  Greg Knapp  Cindy Lester

Discussion Day and Time (circle one): Wed 4pm  Wed 5pm  Thurs 8am  Thurs 9am  Thurs 10am

Instructions: ID Checked (discussion leader only)

Test Materials: During the exam, in addition to a writing tool, you are allowed the use of a scientific calculator (based on the approved list on the syllabus and at your discussion leader’s discretion) as well as a 5” x 8” note card (or half-notebook page) of pre-written notes. Scratch paper will be provided for you during the exam; this scratch as well as your note card will be collected at the end of the exam.

Test-taking Strategies: There are several strategies you can use to best represent your understanding on a successful exam.

- There are 100 possible points and you will have 50 minutes to work. That means you should use at most 0.5 minutes per point a problem is worth (or 12.5 minutes per page) in order to complete the exam in time.

- Glance through the exam at the beginning to see how long it is and what needs to be accomplished.

- If you’re stuck on a problem, move on and come back to it later. Don’t risk forcing yourself to leave exercises blank if you run out of time near the end of the test.

- Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

- Try for partial credit by showing your work where indicated (always possible on free response questions).

Responsible Conduct: Make sure that you are not using any materials other than the ones you have been specifically instructed are allowed, and that you do not communicate with anyone during the exam. Leave your phone and any other communication devices in the classroom if you need to use the restroom during the exam.

Being found responsible for academic misconduct, including communicating with others about exam content before the exams have been returned, is considered cheating and will result in a score of zero on this exam. Sign below to indicate you understand.

Your Signature (required for grading of exam): 

Bonus Code(s) – If you have written down any bonus codes provided by your instructor for extra credit on this exam, write them here: 

_________________________  ________________________
Grading Criteria: Work quality is gauged on accuracy, completeness, and clarity. Specifically, for exercises in which partial credit is available, the following grading rubric will be applied:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Percentage Goal Achievement</th>
<th>Description of Reasoning Demonstrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>A+ (Perfect)</td>
<td>100 Correct and</td>
<td>... all critical reasoning demonstrated clearly</td>
</tr>
<tr>
<td>A (Excellent)</td>
<td>90 – 95 Near correct and</td>
<td>... all critical reasoning demonstrated clearly</td>
</tr>
<tr>
<td>B or B+ (Very Good)</td>
<td>80 – 85 Correct and</td>
<td>... only minor omission of reasoning or</td>
</tr>
<tr>
<td></td>
<td>Nearly correct and</td>
<td>... some omission of reasoning</td>
</tr>
<tr>
<td>C- or C (Passing)</td>
<td>70 – 75 Correct and</td>
<td>... substantial omission of reasoning or</td>
</tr>
<tr>
<td></td>
<td>Mostly correct and</td>
<td>... all reasoning is shown</td>
</tr>
<tr>
<td>D-, D, or D+ (Nearly Passing)</td>
<td>50 – 65 Mostly correct and At most half correct and</td>
<td>... substantial omission of reasoning or</td>
</tr>
<tr>
<td></td>
<td>At most half correct and</td>
<td>... some critical, valid reasoning demonstrated</td>
</tr>
<tr>
<td>F (Does Not Meet Standard)</td>
<td>25 – 40 At most half correct and</td>
<td>... only demonstrated limited valid reasoning</td>
</tr>
<tr>
<td>T (Attempted)</td>
<td>10 – 20 Wholly inaccurate and</td>
<td>... very limited or no valid reasoning demonstrated</td>
</tr>
<tr>
<td>Blank</td>
<td>0 No work provided</td>
<td>No work provided</td>
</tr>
</tbody>
</table>

No points received: The solution writer was caught cheating (using an unauthorized calculator).

<table>
<thead>
<tr>
<th>Page 3</th>
<th>Page 4</th>
<th>Page 5</th>
<th>Page 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points Received</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Points Possible</td>
<td>25</td>
<td>25</td>
<td>16</td>
<td>34</td>
</tr>
</tbody>
</table>

1For cases in which the rubric does not directly apply, the grader will use discretion in determining the most appropriate score.
Multiple Choice and Fill in the Blank Choose the best answer from among the multiple choices given. In each answer blank, write the correct numerical, symbolic expression or phrase. Numerical answers can be expressed exactly or rounded to three decimal places.

1. [5 pts] Multiple Choice: The graph of \( y = g(x) \) is shown to the right. Which of the following expressions has the smallest value?

   (a) \( \int_a^b g(x) \, dx > 0 \) but some values are negative

   (b) \( \int_b^c g(x) \, dx < 0 \)

   (c) \( \int_c^d g(x) \, dx > 0 \) but some values are negative

   (d) \( \int_d^e g(x) \, dx > 0 \) but some values are negative

   (e) \( \int_b^e g(x) \, dx = \int_b^c g(x) \, dx + \int_c^d g(x) \, dx > \int_b^c g(x) \, dx \) since \( \int_c^d g(x) \, dx > 0 \).

2. [10 pts] Fill in the Blank:
   Compute: \( \int \left( 12.4 \, x^{2.1} + \frac{3}{x} + \frac{14}{x^2} - 9 \right) \, dx \)
   \[
   = 4 \cdot x^{3.1} + 3 \ln |x| - 14x^{-1} - 9x + C
   \]
   Work shown (partial credit possible):
   \[
   \frac{12.4}{3.1} x^{3.1} + 3 \ln |x| + \frac{14}{1} x^{-1} - 9x + C
   \]

3. [6 pts] Multiple Choice: A product’s excitement index \( E \) changes at a rate equal to the difference between the purchase rate \( P \) and the sales rate \( S \). Write a differential equation to model this relationship.

   (a) \( E = k(P - S) \)

   (b) \( E = P - S \)

   (c) \( \frac{dE}{dt} = E \)

   (d) \( \frac{dE}{dt} = k(P - S) \)

   (e) \( \frac{dE}{dt} = kt + C \)

   (f) \( \frac{dE}{dt} = kt \)

   (g) \( \frac{dE}{dt} = P - S \)

   (h) \( \frac{dE}{dt} = kE \)

   (i) None of the above

4. [4 pts] Multiple Choice: Suppose the units of a function \( A(x) \) are “hundreds of visitors” and the units of \( x \) are “days”. What are the units of \( \int A(x) \, dx ? \)

   (a) hundreds of days

   (b) hundreds of visitor-days

   (c) days per hundred visitors

   (d) hundreds of visitors per day

   (e) hundreds of visitors

   (f) days

   (g) None of the above.
5. [5 pts] **Multiple Choice:** The function \( H(t) = (4t^2 + 2)^{1/4} + 5 \) is an antiderivative of another function \( h(t) \). What is the formula for \( h(t) \)?

(a) \( h(t) = \frac{4}{20t^3} (4t^2 + 2)^{5/4} + 5t + C \)
(b) \( h(t) = \frac{1}{4}(4t^2 + 2)^{5/4} + 5t + C \)
(c) \( h(t) = \frac{1}{4}(4t^2 + 2)^{3/4} \)
(d) \( h(t) = \frac{1}{4}(4t^2 + 2)^{1/4} \)

(e) \( h(t) = \frac{8}{4}(4t^2 + 2)^{-3/4} \)
(f) \( h(t) = \frac{8}{4}(4t^2 + 2)^{-3/4} + C \)
(g) It is impossible to determine \( h(t) \).
(h) None of the above.

6. [12 pts] **Fill in the Blank:** Find a general solution to the differential equation \( \frac{dy}{dt} = t^3 e^{tx} \).

\[
y = \frac{-1}{2} \ln(C - 3t^4) \quad \text{for some constant } C.
\]

7. [8 pts] **Fill in the Blank:** Use the graph provided below to find the value of \( \int_{-9}^{6} f(t) \, dt \).

\[
\int_{-9}^{6} f(t) \, dt = \frac{6^3}{2} = 31.5
\]
8. [8 pts] **Fill in the Blank:** Given the following:

\[ \int_2^5 R(x) \, dx = 3 \quad \int_5^{10} R(x) \, dx = 4 \]
\[ \int_2^5 N(x) \, dx = -7 \quad \int_5^{10} N(x) \, dx = -6 \]

Evaluate

\[ \int_2^{10} [4R(x) - 6N(x)] \, dx = \frac{106}{4} \]

\[
\begin{align*}
\int_2^{10} [4R(x) - 6N(x)] \, dx &= 4 \int_2^{10} R(x) \, dx - 6 \int_2^{10} N(x) \, dx \\
&= 4 \left[ \int_2^5 R(x) \, dx + \int_5^{10} R(x) \, dx \right] - 6 \left[ \int_2^5 N(x) \, dx + \int_5^{10} N(x) \, dx \right] \\
&= 4 \left[ 3 + 4 \right] - 6 \left[ -7 - 6 \right] = 28 + 78 = 106
\end{align*}
\]

**Free Response** Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written either in exact (unsimplified) or in approximate form as long as an exact solving method is used. Clearly mark your final answers, and include units in all relevant parts.

9. [8 pts] Find the value of \( k \) that makes the expression \( y = 6e^{kx} - 4 \) a solution to the equation \( y' + 5y + 20 = 0 \).

\[
\begin{align*}
y(x) &= 6e^{kx} - 4 \\
y'(x) &= (6k)e^{kx} \\
y'(x) + 5y(x) + 20 &= 6ke^{kx} + 5(6e^{kx} - 4) + 20 \\
&= 6ke^{kx} + 30e^{kx} - 20 + 20 \\
&= (6k + 30)e^{kx}
\end{align*}
\]

We want this to be zero for all \( x \), which happens if and only if \( 6k + 30 = 0 \), that is, \( k = -5 \).
10. Consider the function $v(t) = 12t^2\sqrt{t^3 + 6}$.

(a) [8 pts] Approximate $\int_2^{10} v(t) \, dt$ using a left Riemann sum with four rectangles.

The lengths of the bases are $\frac{10-2}{4} = 2$, so their left endpoints are

$2, 2+2=4, 2+2.2=6, \text{ and } 2+3.2=8$. So:

$[v(2) + v(4) + v(6) + v(8)] \cdot 2 = [12 \cdot 2^2 \sqrt{2^3 + 6} + 12 \cdot 4^2 \sqrt{4^3 + 6} + 12 \cdot 6^2 \sqrt{6^3 + 6} + 12 \cdot 8^2 \sqrt{8^3 + 6}] \cdot 2$

(b) [16 pts] Find the exact value of $\int_2^{10} v(t) \, dt$.

Substitute $u = t^3 + 6$, so $du = 3t^2 \, dt$, and $12t^2 \, dt = 4 \, du$.

Thus $\int v(t) \, dt = \int 12t^2 \sqrt{t^3 + 6} \, dt = \int u^{1/2} \cdot 4 \, du = 4 \int u^{1/2} \, du = 4 \left( \frac{2}{3} \right) u^{3/2} + C$.

Thus $\int_2^{10} v(t) \, dt = \frac{8}{3} \left( (10)^{3/2} - (2)^{3/2} \right) = \frac{8}{3} \left( 10 \sqrt{10} - 2 \sqrt{2} \right) = \frac{8}{3} \left( 10 \sqrt{10} - 2 \sqrt{2} \right) = \frac{8}{3} \cdot 10 \sqrt{10} - \frac{8}{3} \cdot 2 \sqrt{2}$.

(c) [10 pts] Suppose $v(t)$ is the rate of change in market valuation for a house (in yuan per year) $t$ years from now. Write a complete sentence interpreting the computation from part (b) in the applied context, including units.

The value of the house will increase by $\left[ \frac{8}{3} \left( 10 \sqrt{10} \right)^{3/2} - \frac{8}{3} \left( 2 \sqrt{2} \right)^{3/2} \right]$ yuan during the period from two years from now to ten years from now.

[This was written on 18 Feb. 2020, so the value of the house will increase by $\left[ \frac{8}{3} \left( 10 \sqrt{10} \right)^{3/2} - \frac{8}{3} \left( 2 \sqrt{2} \right)^{3/2} \right]$ yuan from 18 Feb. 2022 to 18 Feb. 2030.]