Example 1: Find \( \int \frac{e^x}{1+e^x} \, dx \)

Try substitution: \( v = 1 + e^x \)

Then \( dv = e^x \, dx \)

So \( \int \frac{e^x}{1+e^x} \, dx = \int \frac{dv}{v} \)

\[ = \ln |v| + C \]

\[ = \ln (|1 + e^x|) + C \]

\[ = \ln (1 + e^x) + C \quad \text{since } e^x > 0 \text{ for all real } x. \]
Example 2(a) The rate of change of the population of badgers is proportional to the current population of badgers. Write an equation which expresses this.

Let \( b(t) \) be the population of badgers at time \( t \).

It says: \( b'(t) \) is proportional to \( b(t) \).

That is, there is a constant \( k \) such that, for all times \( t \),

\[
b'(t) = k \cdot b(t)
\]

Example 2(b) Suppose the badgers eat birds, and the rate of change of the population of badgers is proportional to the product of the population of badgers and the population of birds.

Let \( b(t) \) be the population of badgers at time \( t \).

Let \( p(t) \) be the number of birds at time \( t \).

So there is a constant \( R \) such that

\[
b'(t) = R \cdot b(t) \cdot p(t).
\]