Final Exam Overview

Basics

The final exam covers all the material of the course: Sections 5-1–5.5, 6.3, 6.4, and 7.1–7.3.

This study guide covers only Sections 5–1–5.5 and 6.3. Nevertheless, Sections 6.4 and 7.1–7.3 will also be on the final exam. For Sections 7.1–7.3, see the first midterm and its study guides. Information for Section 6.4 will be posted elsewhere.

Because of the unusual circumstances of this exam, problem types may be a bit different from those given here.

Skills List

The following is list of skills you will need to be able to apply on the exam, from Sections 5–1–5.5 and 6.3 only. This list is intended to be comprehensive, whereas the practice problems and those done in class are only a sampling. By its nature it is somewhat more vague than a sampling of practice problems. I have attempted to pair each exercise with a relevant example to illustrate it. This is no guarantee that a particular example will embody the approach to that skill on the exam and in almost all cases no single example sufficiently addresses all possible such uses of the given skill.

With the aid of a scientific calculator and relevant formulas, you should be able to …

1. Find a family of antiderivatives using the power rule, $x^{-1}$, and the rule for exponential functions.

   **Example:** Compute $\int \frac{e^{3x} - e^{-x}}{e^x} \, dx$.

2. Use sum, difference, and constant multiple rules of the indefinite and definite integrals.

   **Example:** If $\int f(x) \, dx = x \ln(x) + C_1$, and $\int g(x) \, dx = \sqrt{x^2 + 1} + C_2$, then compute $\int [2f(x) - 3g(x)] \, dx$.

3. Evaluate definite integrals involving the property that for $a < c < b$, we have $\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$.

   **Example:** Assuming that $\int_{-3}^0 f(x) \, dx = 4$ and $\int_{-3}^2 f(x) \, dx = -2$, find the value of $\int_0^2 f(x) \, dx$.

4. Set up a differential equation based on a written description.

   **Example:** The rate of change (with respect to the distance, $r$, from the center of a city) in density, $D$, of individuals in a metropolitan area is inversely proportional to the density of individuals, with constant of proportionality equal to $-3$. 
5. Verify a provided solution to a differential equation.

**Example:** Show that \( X = Ce^t + t + 4 \) is a solution to the differential equation
\[
X - \frac{dX}{dt} = t + 3
\]
where \( C \) is any constant.

6. Find a general or particular solution to a simple differential equation by integration.

**Example:** Find the general solution to the equation \( \frac{dx}{dt} = \frac{19}{t} - 19t^2 \).

7. Find a general or particular solution to a separable differential equation by integration.

**Example:** The rate of change (with respect to the distance, \( r \), from the center of a city) in density, \( D \), of individuals in a metropolitan area is inversely proportional to the density of individuals, with constant of proportionality equal to \(-3\). Find a formula for the density as a function of \( r \), given that the density is 10 thousand individuals per square mile at a distance of 1 mile from the center of the city.

8. Find a family of antiderivatives using substitution.

**Example:** Compute \( \int (2t + 1)e^{t^2 + t - 5} \, dt \).

9. Use a provided formula for the antiderivative of \( xe^{kx} \).

**Example:** Find the family of antiderivatives of \( H(x) = -3xe^{0.1x} \).

10. Interpret the definite integral as the (signed) area under a curve.

**Example:** Find the area of the region contained between the graph of \( y = e^{-x} - 1 \), the \( x \)-axis, and the line \( x = -1 \).

11. Compute an indicated Riemann sum.

**Example:** Approximate \( \int_2^8 f(t) \, dt \) using a left Riemann sum with 3 subdivisions, given the graph of \( y = f(t) \) provided.

12. Find the value of a definite integral using the Fundamental Theorem of Calculus.

**Example:** Compute \( \int_{-2}^{2} \left( \frac{2}{t+3} - 1 \right) \, dt \).
13. Interpret the definite integral of a marginal function as a net change in the original function.  

Example: Shares of a stock are traded at a rate approximated by \( S'(t) = 3\sqrt{t} \) thousand shares per day, \( t \) days after the beginning of the year. How many shares are traded during January?

14. Interpret the area of the region between two curves as a definite integral.  

Example: Compute the net excess profit for the investments having profit equal to \( A(t) \) and \( B(t) \), respectively, with \( t \) in years, over the course of the year after the first. Assume that \( A'(t) = \frac{10}{t} \) and \( B'(t) = \frac{10}{t^2} \) thousand dollars per year.

15. Compute and reason regarding income inequality using Lorenz curves and the Gini Index.  

Example: Compute the Gini Index for a group having Lorenz curve \( L(x) = \frac{4}{5}x^2 + \frac{1}{5}x \).

16. Find the average value of a function on a specified interval.  

Example: The rate of production of an oil field is given by \( R(t) = \frac{10t}{t^2 + 1} \) hundred thousand barrels per year, \( t \) years from initial extraction. What is the average rate of yearly production over the 20-year lifetime of the oil field?

17. Find the future or present value of a continuous income stream.  

Example: Find the present value of the income stream which is ten thousand dollars per year initially, then fifteen thousand at the beginning of the the second, twenty thousand at the beginning of the third, and so on increasing linearly over time. Assume that the income stream is invested at 2% interest compounded continuously exists in perpetuity.

18. Compute definite integrals using additional economics or business terminology.  

Example: Government regulation holds the price of a key commodity constant at $5 per pound. The rate of change in daily demand for the commodity \( t \) weeks after the beginning of the year is given by \( D'(t) = \frac{12}{(t + 2)^2} \) thousand pounds per week, while the rate of change in total daily cost to extract and sell the commodity is \( C'(t) = 8 - 0.3t \) thousand dollars per week. By how much does daily profit change in the first ten weeks of the year?

19. Find the consumer or producer surplus in a market.  

Example: The total market surplus is the sum of consumer and producer surplus. Find the total market surplus at the equilibrium for \( D(q) = 50e^{-2q} \) and \( S(q) = 20e^q \), where \( q \) is thousands of units sold and each function gives price in dollars per unit.
20. Evaluate an improper integral and classify it as either convergent or divergent.

Example: Identify to what value the integral
\[ \int_{5}^{\infty} \frac{3}{t^{0.9}} \, dt \] converges or state that it diverges.
Additional Free Response-Style Questions

In Exercises #21 – #24, use the fact that the marginal profit for the company Trees Inc. when processing \( x \) tons of lumber per day is \( P'(x) = 90 - 120e^{-0.2x} \) dollars per ton.

21. What is the net change in Trees Inc.’s total profit when their production rises from 10 to 20 tons of lumber per day?

22. What is Trees Inc.’s average marginal profit for production levels between 5 and 10 daily tons?

23. If the production level for Trees Inc. were increased without bound, what would the change in total profit for Trees Inc. converge to, or would it diverge?

24. Up and coming competitor Log Co. has a marginal profit of \( Q'(x) = 120 - 240e^{-0.2x} \). CEO of Trees Inc. plans to retire once Log Co. reaches a production level at which Log Co.’s marginal profit is greater than his company’s. What is the net excess profit for Trees Inc. over Log Co. for production levels up to that point?

25. A country has Lorenz curve \( L(x) = ax^2 + (1 - a)x^3 \), for some constant \( a \)

(a) Census data shows that the poorest half of the population controls 30% of the group’s wealth. Use this information to find the value of \( a \).

(b) Find the Gini index for the group.

26. What lump sum investment now is necessary to match a continuously-deposited 100,000 dollars per year income stream if each is invested at a continuous annual rate of 4% for fifteen years?

27. A quantity adjustment model specifies that the rate of change (with respect to time) in the hourly quantity of a product sold is proportional to the difference between the demand and supply. For a particular product, the hourly demand is \( d = 12 - q \) dollars per item whereas hourly supply is \( s = q + 2 \) dollars per item, when \( q \) hourly units are sold.

(a) Initially, the hourly quantity sold is 7, and is decreasing at a rate of $1 per day. Write a differential equation corresponding to the description above, including finding the value of any constant of proportionality.

(b) Use this information to find a formula for the quantity sold \( t \) days from now.