Instructions:

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you’re stuck on a problem, move on and come back to it later. Don’t risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. The exam is worth 50 points. That means you should spend around 1.6 minutes for each point the problem is worth in order to complete the exam in time.

Partial credit is possible. Any fill blank or multiple choice items with space left for “work shown (partial credit possible)” can receive up to half credit for the work shown. Partial credit is always available on free response questions. In the limited space provided, be careful to only include what you want your instructors to evaluate.

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Bonus Codes:
Multiple Choice and Fill Blank Choose the best answer from among the multiple choices given. In each answer blank, write the correct numerical, symbolic expression or phrase (e.g. “DNE”). Numerical answers can be expressed exactly or rounded to three decimal places.

1. **Multiple Choice:** Which is the integration step in solving the differential equation \( \frac{dP}{dx} = x^2 \cdot (P^3 + 1) \)?

2. Consider the continuous income stream with deposits of size \( f(t) = 500e^{0.04t} \) dollars per month, earning 0.2% monthly interest.

   (a) **Fill Blank:** Find the future value of the continuous income stream over the course of two years. (Careful with units)

   / 4 Value: ____________________

   (b) **Multiple Choice:** What happens to the future value as the length of time is increased without bound?

   / 2
   i. It diverges toward \( \infty \).
   ii. It diverges toward \( -\infty \).
   iii. It converges to a value less than 5 million dollars.
   iv. It converges to a value greater than 5 million dollars.

3. **Fill Blank:** The graphs of linear supply and demand functions are given, \( p = S(q) \) and \( p = D(q) \), respectively, both measured in thousands of yen per item when \( q \) items are sold.

   Compute the producer surplus at market equilibrium (including units).

   / 4 Surplus: ____________________

   Work shown (partial credit possible)
4. **Fill Blank:** Find the area of the region contained between the curves defined by \( y = x^2 - 6 \) and \( y = -1x \).

\[
\text{Area: } \quad \frac{1}{4} 
\]

5. **Fill Blank:** The likelihood that the next accident occurs in a warehouse after \( x \) weeks is given by the probability density function \( P(x) = 0.4e^{-0.4x} \) for \( x \geq 0 \) and 0 otherwise. What is the probability that the next accident occurs after at least 4 weeks?

\[
\text{Probability: } \quad \frac{1}{4} 
\]

6. The rate of change in market share for a tech development company is given by \( S(t) = \frac{60}{t + 4} - 15 \) percent per year, \( t \) years from now.

(a) **Fill Blank:** What is the average value of the rate of change in market share over the next four years?

\[
\text{Value: } \quad \frac{1}{4} 
\]

(b) **Multiple Choice:** Which expression represents the net change in market share over the next four years?

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\text{Work shown (partial credit possible)}
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\text{Work shown (partial credit possible)}
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\text{Work shown (partial credit possible)}
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7. **Multiple Select:** Circle any and all of the following which must be true of a Lorenz curve, \( L(x) \), describing the portion of income controlled by the poorest fraction \( x \) of a group.
   - \( L(x) \) is defined only for \( 0 \leq x \leq 1 \).
   - \( L'(x) > 0 \) for all values of \( x \) in the domain.
   - \( 0 \leq L(x) \leq 1 \) for all values of \( x \) in the domain.
   - \( L(x) > x \) for all values of \( x \) in the domain.

8. A random variable is described by the probability density function \( f(x) = \begin{cases} \frac{k}{x^2}, & \text{if } 1 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases} \)

(a) **Fill Blank:** Find the value of \( k \).

(b) **Multiple Choice:** Write the expected value of the random variable.

9. **Fill Blank:** For what value of the constant \( A \) is \( y = e^{-x} + 4x^2 + A(1 - x) \) a solution to the differential equation \( y' + y = 4x^2 \)?

10. **Fill Blank:** Use a left Riemann sum with \( n = 4 \) to approximate the value of \( \int_{-2}^{6} f(x) \, dx \) for the function \( y = f(x) \) shown in the graph below.
Free Response Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written either in exact (unsimplified) or in approximate form as long as an exact solving method is used. Clearly mark your final answer, and include units in all relevant parts.

11. An insurance company offers retirement plans which pay out to customers continuously at a rate of $30000 per year at \( t = 0 \), $34000 per year at \( t = 1 \), and so on increasing linearly each year and invested continuously at an annual rate of 8%. Assuming an annual interest rate of 8% on the investment, compounded continuously, how much would the company need to invest as a lump sum right now in order to support a customer’s retirement plan in perpetuity?

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12. Marginal propensity to save, \( S(I) \), is the rate of change (with respect to a person’s income, \( I \)) in savings. Assume that both income and savings are measured in thousands of dollars. Write a sentence interpreting the equation \( \int_{40}^{70} M(I) \, dI = 8 \) in terms of income and/or savings.

\[
/M/ 4
\]