Final Exam
Math 242, Winter ’18va

Your Name: ____________________________  Student Number: ____________________________

Your Discussion Leader's Name (circle one):

Dana Fry   Mike Gartner   Eli Hulbert

Your Discussion Day and Time (circle one):

Wed 4pm    Wed 5pm    Thurs 9am    Thurs 10am

Instructions:

ID Checked (discussion leader only)

Check your answers. Take the time before you turn in your test to make sure you have read the directions
correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your
answer (by boxing or circling it where appropriate).

Pace yourself. If you’re stuck on a problem, move on and come back to it later. Don’t risk forcing yourself to
give partial answers if you run out of time near the end of the test. Do the easy ones first. The exam is worth
40 points. That means you should spend around 3.0 minutes for each point the problem is worth in order to
complete the exam in time.

Try for partial credit. Any fill blank or multiple choice items with space left for “work shown (partial credit
possible)” can receive up to half credit for the work shown. Partial credit is always available on free response
questions. In the limited space provided, be careful to only include what you want your instructors to evaluate.

Be ethical. Provide your own work on the exam, without cheating. Being found responsible for academic
misconduct, including communicating with others during the exam, is considered cheating and will result in a
score of zero on this exam. Sign below to indicate you understand.

Your Signature (required for grading of exam): ____________________________________________

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Bonus Codes:

_________________________________  __________________________________
Multiple Choice and Fill Blank  Choose the best answer from among the multiple choices given. In each answer blank, write the correct numerical, symbolic expression or phrase (e.g. “DNE”). Numerical answers can be expressed exactly or rounded to three decimal places.

Omit Exercise #1, we did not cover Section 6.4

1. Consider the probability density function 
   \[ f(x) = \begin{cases} 
   \frac{kx}{x^2 + 2} & \text{if } 1 \leq x \leq 4 \\
   0 & \text{otherwise} 
   \end{cases} \]
   and where \( k \) is an unknown constant.

   (a) **Fill Blank:** Find the value of \( k \) so that 
   \( f(x) \) is a probability density function.
   \( k = \frac{1}{4} \)

   (b) **Multiple Choice:** Which expression provides the expected value of \( X \)?
   \[ \begin{align*}
   &\text{i. } \int_{1}^{4} \frac{k}{x^2 + 2} \, dx \\
   &\text{ii. } \int_{-\infty}^{1} \frac{k}{x^2 + 2} \, dx \\
   &\text{iii. } \int_{-\infty}^{4} \frac{kx^2}{x^2 + 2} \, dx \\
   &\text{iv. } \int_{-\infty}^{4} \frac{kx}{x^2 + 2} \, dx \\
   &\text{v. } \int_{-\infty}^{\infty} \frac{k}{x^2 + 2} \, dx \\
   &\text{vi. } \int_{-\infty}^{\infty} \frac{kx}{x^2 + 2} \, dx 
   \end{align*} \]

2. **Fill Blank:** Julio places his entire savings of \( D \) dollars into an account earning 0.1% interest, compounded continuously. At the same time Marissa begins to invest a continuous income stream of $16,000 per year at the same interest rate as Julio’s account. At the end of eight years, their accounts have the same value. What is the value of \( D \)?

   \( D = \$ \) __________

3. **Multiple Choice:** Which of the following gives the integration step in solving the equation \( \frac{dy}{dt} = \frac{y}{t^2} \)?

   \[ \begin{align*}
   &\text{(a) } \int \frac{dy}{dt} = \int \frac{y}{t^2} \\
   &\text{(b) } \int \frac{1}{y} \, dy = \int \frac{1}{t^2} \, dt \\
   &\text{(c) } \int y \, dy = \int t^2 \, dt \\
   &\text{(d) } \int t^2 \, dy = \int y \, dt \\
   &\text{(e) } \int dy = \int \frac{y}{t^2} \, dt \\
   &\text{(f) } \int \frac{1}{y} \, dy = \int t^2 \, dt 
   \end{align*} \]
4. Consider the function \( y = Q(x) \) depicted below in solid line segments and \( y = M(x) \) as a dashed line.

(a) **Multiple Choice:** Which expression represents the area of the region contained completely by the two curves \( M(x) \) and \( Q(x) \)?

\[
\begin{align*}
&i. \quad \int_0^7 [M(x) - Q(x)] \, dx \\
&ii. \quad \int_7^1 [Q(x) - M(x)] \, dx \\
&iii. \quad \int_7^1 [Q(x) - M(x)] \, dx \\
&iv. \quad \int_1^7 [M(x) - Q(x)] \, dx \\
&v. \quad \int [Q(x) - M(x)] \, dx \\
&vi. \quad \int [M(x) - Q(x)] \, dx
\end{align*}
\]

(b) **Fill Blank:** Find the average value of \( Q(x) \) on the interval \([2, 7]\).

\[
AV = \quad \text{___________}
\]

5. **Fill Blank:** As measured by the Gini Index, what is the inequality of Group B, whose Lorenz curve is \( B(x) = 0.1x + 0.90x^2 \)? Does that represent more or less inequality than Group D with Gini Index 0.35? Circle “more” or “less”.

Group B’s Gini Index is __________, which represents more/less inequality than Group D.

6. **Multiple Choice:** The function \( y = -5e^{2x} \) is a solution to which differential equation?

\[
\begin{align*}
&i. \quad y' = -5y \\
&ii. \quad y' = 2y \\
&iii. \quad y' = 5y \\
&iv. \quad y' = -2y
\end{align*}
\]

7. **Multiple Select:** Suppose that \( N(x) \) is an antiderivative of \( H(x) \) and \( C \) is an arbitrary constant. Then which two of the following statements must be true?

\[
\begin{align*}
&(a) \quad N'(x) = H(x) \quad (b) \quad H(x) = N(x) \quad (c) \quad H'(x) = N(x) \quad (d) \quad \int H(x) = N(x) + C \quad (e) \quad \int N(x) = H(x) + C
\end{align*}
\]
Free Response Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written either in exact (unsimplified) or in approximate form as long as an exact solving method is used. Clearly mark your final answers, and include units in all relevant parts.

8. The rate of change in number of accounts on a dating app is \( r(t) = 12te^{-0.1t} \) hundred accounts per week, \( t \) weeks after the beginning of the year. What is the net change in number of accounts starting at three weeks after the beginning of the year and into the long run?

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Omit Exercise #9, we did not cover Section 6.4

9. The length of time an employee serves as shift manager is a random variable \( M \) with an exponential distribution and expected value 8 months. What is the probability that an employee is shift manager for between five and nine months?

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10. At a price of \( p \) euro per kilogram, \( q \) thousand kilograms of a particular commodity are sold. Demand for the commodity is \( p = \frac{8}{q^2 + 0.5} \) while supply is \( p = 2 - \frac{1}{q^2 + 0.5} \). Use a left Riemann sum with five rectangles to approximate the producers’ surplus at the market equilibrium of 2 thousand kilograms sold at 1.78 euro per kilogram.

11. The marginal cost for production of wheat is \( 150 + 4000(x + 2)^{-2} \) dollars per acre. Find a formula for the total cost to produce \( x \) acres of wheat when the overhead (fixed cost) is $6,000.