WORKSHEET SOLUTIONS: PRODUCT AND QUOTIENT RULES

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the quotient rule: if f and g are differentiable, $g(x)$ is never zero (on a suitable open interval) and

$$j(x) = \frac{f(x)}{g(x)}$$

for all x (in a suitable open interval), then

$$j'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

1. Differentiate the function $j(x) = \dfrac{\sin(x)}{x^2 + x + 78}$.

Solution. Write

$$j(x) = \frac{f(x)}{g(x)} \qquad \text{with} \qquad f(x) = \sin(x) \qquad \text{and} \qquad g(x) = x^2 + x + 78.$$

Thus:

$$\begin{aligned} j'(x) &= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} = \frac{\sin'(x)(x^2 + x + 78) - \sin(x)\dfrac{d}{dx}\big(x^2 + x + 78\big)}{(x^2 + x + 78)^2} \\ &= \frac{(x^2 + x + 78)\cos(x) - (2x + 1)\sin(x)}{(x^2 + x + 78)^2}. \end{aligned}$$

If your solution contains "$\cancel{2x + 1\sin(x)}$", it is wrong: the parentheses are essential. □

2. Differentiate the function $q(x) = \dfrac{x^3 \sin(x)}{x^2 + x + 78}$. You will need both the product and quotient rules. To decide which one to use first, look at which operation is done **last** when evaluating, say, $q(2)$.

Solution. Use the quotient rule, using the product rule where one needs to differentiate the numerator. That is, write

$$j(x) = \frac{f(x)}{g(x)} \qquad \text{with} \qquad f(x) = x^3 \sin(x) \qquad \text{and} \qquad g(x) = x^2 + x + 78.$$

Thus:

$$\begin{aligned} q'(x) &= \frac{\dfrac{d}{dx}\big(x^3 \sin(x)\big)(x^2 + x + 78) - x^3 \sin(x)\dfrac{d}{dx}\big(x^2 + x + 78\big)}{(x^2 + x + 78)^2} \\ &= \frac{\big[3x^2 \sin(x) + x^3 \cos(x)\big](x^2 + x + 78) - x^3 \sin(x) \cdot (2x + 1)}{(x^2 + x + 78)^2} \\ &= \frac{\big[3x^2 \sin(x) + x^3 \cos(x)\big](x^2 + x + 78) - (2x^4 + x^3)\sin(x)}{(x^2 + x + 78)^2} \\ &= \frac{\big(x^4 + 2x^3 + 3 \cdot 78x^2\big)\sin(x) + x^3(x^2 + x + 78)\cos(x)}{(x^2 + x + 78)^2}. \end{aligned}$$

□

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3. Differentiate the function $f(x) = \dfrac{2}{x^3}$. Do **not** use the quotient rule!

Solution. Write $f(x) = 2x^{-3}$. So $f'(x) = 2 \cdot (-3)x^{-3-1} = -6x^{-4}$. □

4. Differentiate the function $g(x) = \sqrt[3]{x}$.

Solution. Write $g(x) = x^{1/3}$. So $g'(x) = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3}$. □

5. Differentiate the function $h(x) = \dfrac{1}{\sqrt[3]{x}}$. Do **not** use the quotient rule!

Solution. Write $h(x) = x^{-1/3}$. So $h'(x) = -\frac{1}{3}x^{-1/3-1} = -\frac{1}{3}x^{-4/3}$. □