WORKSHEET SOLUTIONS: FINDING THE ABSOLUTE MINIMUM AND MAXIMUM 2

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the method for maximizing and minimizing a continuous function f on a closed interval $[a, b]$:

(1) Find all the critical numbers c of f in $[a, b]$, that is, numbers c in $[a, b]$ such that $f'(c) = 0$ or $f'(c)$ does not exist.
(2) Evaluate f (**not** f'!) at all the numbers c found in (1) and at the endpoints a and b.
(3) The absolute maximum of f on $[a, b]$ is the largest value you got in (2). It occurs at the value of x you used to produce it. (In rare cases, the absolute maximum can occur at more than one value of x.) Similarly, the absolute minimum of f on $[a, b]$ is the smallest value you got in (2), etc.

Let $f(x) = x^3 - 6x^2 + 16$. Use the methods of calculus to find the exact values of x (*not* calculator approximations) at which f has its maximum and minimum values on the interval $[-1, 2]$.

The problem does not ask you to find the exact maximum and minimum values of f on the interval, only the exact values of x at which they occur.

1. Find all critical numbers of f.

Solution. The derivative of f is

$$f'(x) = 3x^2 - 12x = 3x(x - 4).$$

It exists everywhere, and it is zero when $x = 0$ and $x = 4$. Therefore the critical points are $c = 0$ and $c = 4$. □

2. Which critical numbers are in $[-1, 2]$?

Solution. Only 0 is in $[-1, 2]$. □

3. List the values of x at which one should evaluate f according to Step (2) at the top of the page.

Solution. Use $x = -1$, $x = 2$ (the endpoints), and $x = 0$ (the critical number in $[-1, 2]$). □

4. Evaluate f at each of the numbers just found. No calculator should be needed. Remember that you won't have one on the exams. (Do **not** evaluate f' at those numbers! But **do** explain why it is not helpful to do that.)

Solution. I got

$$f(-1) = (-1)^3 - 6(-1)^2 + 16 = -1 - 6 + 16 = 9, \qquad f(0) = 16,$$

and

$$f(2) = 2^3 - 6 \cdot 2^2 + 16 = 8 - 6 \cdot 4 + 16 = 8 - 24 + 16 = 0.$$

We need the values of f because we are trying to decide at which values of x the function f has its largest and smallest values.

In any case, we already know that $f'(0) = 0$. After all, that is how we found the number 0. □

5. Where does f take its absolute minimum and maximum values on $[-1, 2]$?

Solution. Since, from the previous calculation, $f(2) < f(-1) < f(0)$, the absolute maximum value occurs at $x = 0$ and the absolute minimum value occurs at $x = 2$. □

The value $f(4) = -16$ is smaller than any of these, but we don't use it since 4 is not in the interval $[-1, 2]$.

Date: 21 April 2025.