WORKSHEET: SQUEEZE THEOREM AND TRIGONOMETRIC DERIVATIVES

Names and student IDs: _____

Recall the Squeeze Theorem: if $f(x) \leq g(x) \leq h(x)$ on an open interval containing a, except at a itself, $\lim_{x\to a} f(x) = L$, and $\lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$. In particular, $\lim_{x\to a} g(x)$ exists.

1. Does the Squeeze Theorem apply to $\lim_{x\to 0} x^4 \sin\left(\frac{1}{x^2}\right)$? If so, what do you take for $f(x)$, $g(x)$, and $h(x)$? If not, why not?

2. Does the Squeeze Theorem apply to $\lim_{x\to 0} (1+x^4) \sin\left(\frac{1}{x^2}\right)$? If so, what do you take for $f(x)$, $g(x)$, and $h(x)$? If not, why not?

3. Does the Squeeze Theorem apply to $\lim_{x\to 0} x^4 \left(1 + \sin\left(\frac{1}{x^2}\right)\right)$? If so, what do you take for $f(x)$, $g(x)$, and $h(x)$? If not, why not? (Be careful!)

Recall the derivatives of $\tan(x)$ and $\sec(x)$: $\tan'(x) = \sec^2(x)$ and $\sec'(x) = \sec(x)\tan(x)$.

4. Find $\frac{d}{dx}\big(\tan(x^3 + 5x)\big)$.

5. Find $\frac{d}{dt}\big(\sec(8t)\big)$.

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