WORKSHEET: INFINITE LIMITS

Names and student IDs: _____

Recall: We say $\lim_{x\to a} f(x) = \infty$ (even though ∞ is not a number) if one can force $f(x)$ to be as large as one wants by requiring that x be close enough to a. In particular, f has a vertical asymptote at $x = a$, near which the graph goes up on both sides. Example: $\lim_{x\to o} \frac{1}{x^2} = \infty$.

Plot this on your calculator to see, moving the screen around to show the parts of the function near $x = 0$.

Also look at the following table of values. You see that when x is close to zero (with either $x > 0$ or $x < 0$), then x^2 is positive and close to zero, and $\frac{1}{x^2}$ is ;positive and large.

x	x^2	$\frac{1}{x^2}$
1	1	1
0.1	0.01	100
0.01	0.0001	10,000
0.0001	0.00000001	100,000,000
-1	1	1
-0.1	0.01	100
-0.01	0.0001	10,000
-0.0001	0.00000001	100,000,000

Similar considerations give meanings to $\lim_{x\to a} f(x) = -\infty$, $\lim_{x\to a^-} f(x) = \pm\infty$, $\lim_{x\to\infty} f(x) = \pm\infty$, etc.

1. What are the following? Try computing some values if needed. After you have answered, look at calculator graphs to check.

a. $\lim_{x\to 0} \left(-\frac{1}{x^2}\right)$? (Parentheses are needed in this expression.)

b. $\lim_{x\to 0^-} \frac{1}{x}$?

c. $\lim_{x\to 0^-} \left(\frac{1}{x} + 45\right)$? (Parentheses are needed in this expression.)

Continued on back.

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d. $\lim_{x \to 2^+} \frac{3}{x-2}$?

e. $\lim_{x \to 2^+} \frac{x+1}{x-2}$? (Hint: The function $f(x) = x + 1$ is continuous, and $f(2) = 3$. Compare with the previous problem.)

f. $\lim_{x \to -\infty} (7x^2 + 1)$?

2. Suppose $\lim_{x \to 3} f(x) = \infty$ and $\lim_{x \to 3} g(x) = \infty$. What can you say about $\lim_{x \to 3} (f(x) + g(x))$ and $\lim_{x \to 3} (f(x) - g(x))$?