WORKSHEET SOLUTIONS: TANGENT LINES AND LINEAR APPROXIMATION

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall: the tangent line to the graph of $y = f(x)$ at $x = a$ (at the point $(a, f(a))$ on the graph) goes through the point $(a, f(a))$ and has slope $f'(a)$.

1. Consider the function $g(x) = x^2 + 2x$.

(a) Write the equation of the tangent line to the graph at $x = 2$ in the first form discussed in lecture: $y - y_0 = m(x - x_0)$ for suitable x_0, y_0, m. (This is sometimes called "point-slope" or "slope-point" form.)

Solution. Take $x_0 = 2$. Take $y_0 = g(2) = 8$. Take $m = g'(2)$. We have $g'(x) = 2x + 2$, so $m = g'(2) = 6$. Thus the equation is $y - 8 = 6(x - 2)$. □

(b) Make sure your answer above really is the equation of a line!

Solution. Yes, for $y - 8 = 6(x - 2)$. No, for something like $\cancel{y - 8 = (2x + 2)(x - 2)}$. □

(c) Write the equation of the tangent line to the graph at $x = 2$ in the linear approximation form.

Solution. The linear approximation form at $x = a$ is $L_a(x) = f(a) + f'(a)(x - a)$. So: $L_a(x) = 8 + 6(x - 2)$, or $y = 8 + 6(x - 2)$.

As we will see, for our purposes, this form is the most useful. □

(d) Write the equation of the tangent line to the graph at $x = 2$ in the usual form $y = mx + b$ for some m and b. (I think this is sometimes called "slope-intercept" form.)

Solution. Rewrite $y = 8 + 6(x - 2) = 8 + 6x - 12 = 6x - 4$. Thus: $y = 6x - 4$. □

2. Suppose $f(3) = 9$ and you know f is continuous at 9, but nothing more. What are your best guesses for $f(3.3)$ and $f(2.9)$?

Solution. Your best guess for both is just $f(3.3) \approx 9$ and $f(2.9) \approx 9$. **Do not write** $\cancel{f(3.3) = 9}$ or $\cancel{f(2.9) = 9}$: these are not true! □

3. In problem 2, suppose you **also** know that f is differentiable at 3 and $f'(3) = 2$. Now what are your best guesses for $f(3.3)$ and $f(2.9)$?

Solution. The linear approximation is

$$f(x) \approx L(x) = f(3) + f'(3)(x - 3) = 9 + 2(x - 3).$$

So

$$f(3.3) \approx 9 + 2(3.3 - 3) = 9.6 \qquad \text{and} \qquad f(2.9) \approx 9 + 2(2.9 - 3) = 8.8.$$

Do not write $\cancel{f(x) = L(x)}$, $\cancel{f(3.3)) = 9.6}$, or $\cancel{f(2.9) = 8.8}$: these are not true!

Also, don't rewrite the linear approximation in the form $y = mx + b$. This just makes the arithmetic harder. □

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