WORKSHEET: L'HOPITAL'S RULE

Names and student IDs: _____

Do NOT confuse L'Hopital's Rule with the quotient rule!

Before using L'Hopital's Rule, you must check that its hypotheses are satisfied!

A special case: **if** $\lim_{x\to a} f(x) = 0$ **and** $\lim_{x\to a} g(x) = 0$**,** and if $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

(Generally, one must have a fraction with an **indeterminate form**, such as "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$".)

1. Consider $\lim_{x\to 0} \frac{e^{2x}-1}{\sin(3x)}$. In the above, we will take $f(x) = e^{2x} - 1$ and $g(x) = \sin(3x)$.

(a) What is $\lim_{x\to 0}(e^{2x} - 1)$?

(b) What is $\lim_{x\to 0} \sin(3x)$?

(c) Does $\lim_{x\to 0} \frac{e^{2x}-1}{\sin(3x)}$ have an indeterminate form?

(d) Find $\lim_{x\to 0} \frac{e^{2x}-1}{\sin(3x)}$.

2. Differentiate the function $f(x) = \frac{e^{2x}-1}{\sin(3x)}$.

Continued on back.

Date: 4 June 2025.

3. Consider $\lim_{x\to 0} \frac{x}{x+2}$.

(a) What is $\lim_{x\to 0} x$?

(b) What is $\lim_{x\to 0}(x+2)$?

(c) Does $\lim_{x\to 0} \frac{x}{x+2}$ have an indeterminate form?

(d) Find $\lim_{x\to 0} \frac{x}{x+2}$.

A different special case: **if** $\lim_{x\to\infty} f(x) = \pm\infty$ **and** $\lim_{x\to\infty} g(x) = \pm\infty$**,** and if $\lim_{x\to\infty} \frac{f'(x)}{g'(x)}$ exists, then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}.$$

(Generally, one must have a fraction with an **indeterminate form**, such as "$\frac{0}{0}$" or "$\frac{-\infty}{\infty}$".)

4. Consider $\lim_{x\to\infty} \frac{x}{e^{8x}+3x}$.

(a) What should $f(x)$ and $g(x)$ be?

(b) What is $\lim_{x\to\infty} f(x)$?

(c) What is $\lim_{x\to\infty} g(x)$?

(d) Does $\lim_{x\to\infty} \frac{x}{e^{8x}+3x}$ have an indeterminate form?

(e) Find $\lim_{x\to\infty} \frac{x}{e^{8x}+3x}$.